

# The Precipitation Rate Retrieval Algorithms for the GPM Dual-frequency Precipitation Radar

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205–237, 2021. <http://doi.org/10.2151/jmsj.2021-011>

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1. Introduction

2. Framework

Included in today's talk

3. Scattering table

3.1 Liquid-phase precipitation

3.2 Mixed-phase precipitation

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4. R-Dm relation

5. Non-uniform beam filling

6. Single-frequency algorithms

6.1 Range bin classification (rain possible/no-rain)

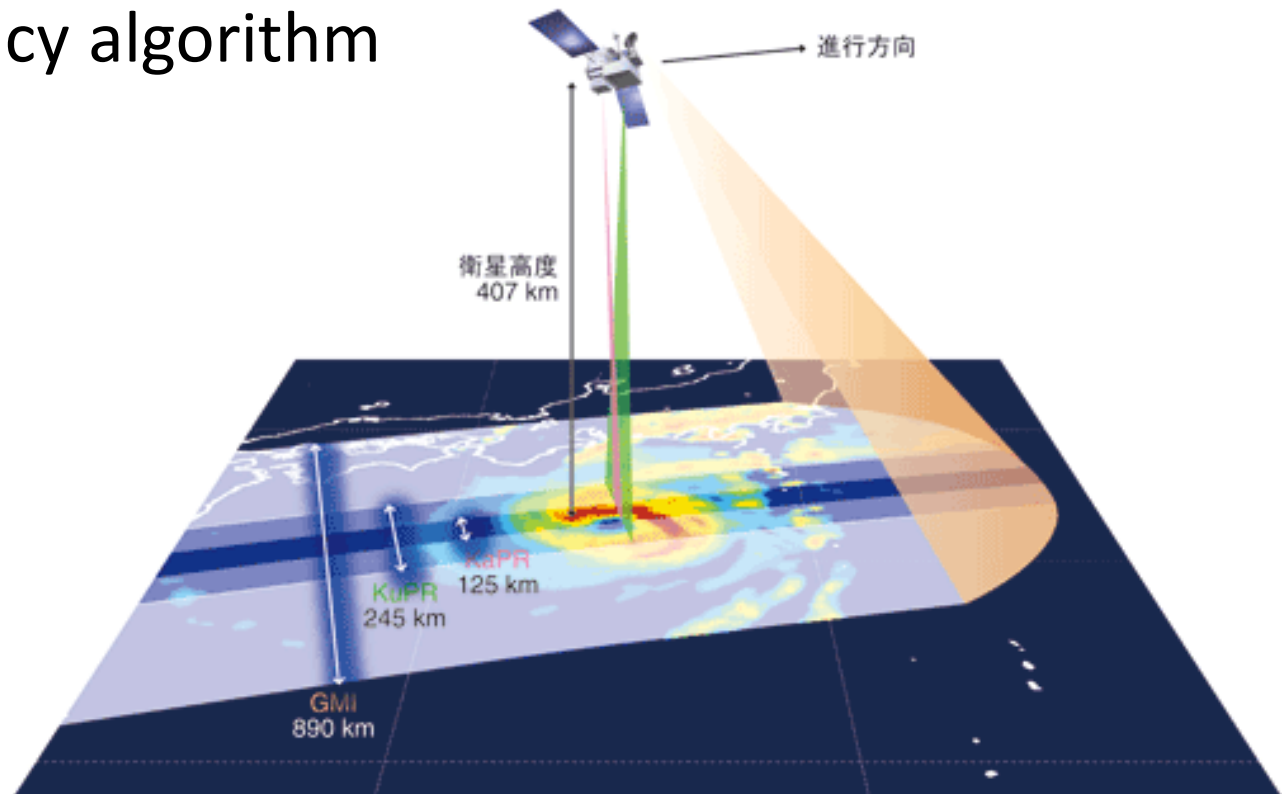
6.3 DSD database

7. Dual-frequency algorithm

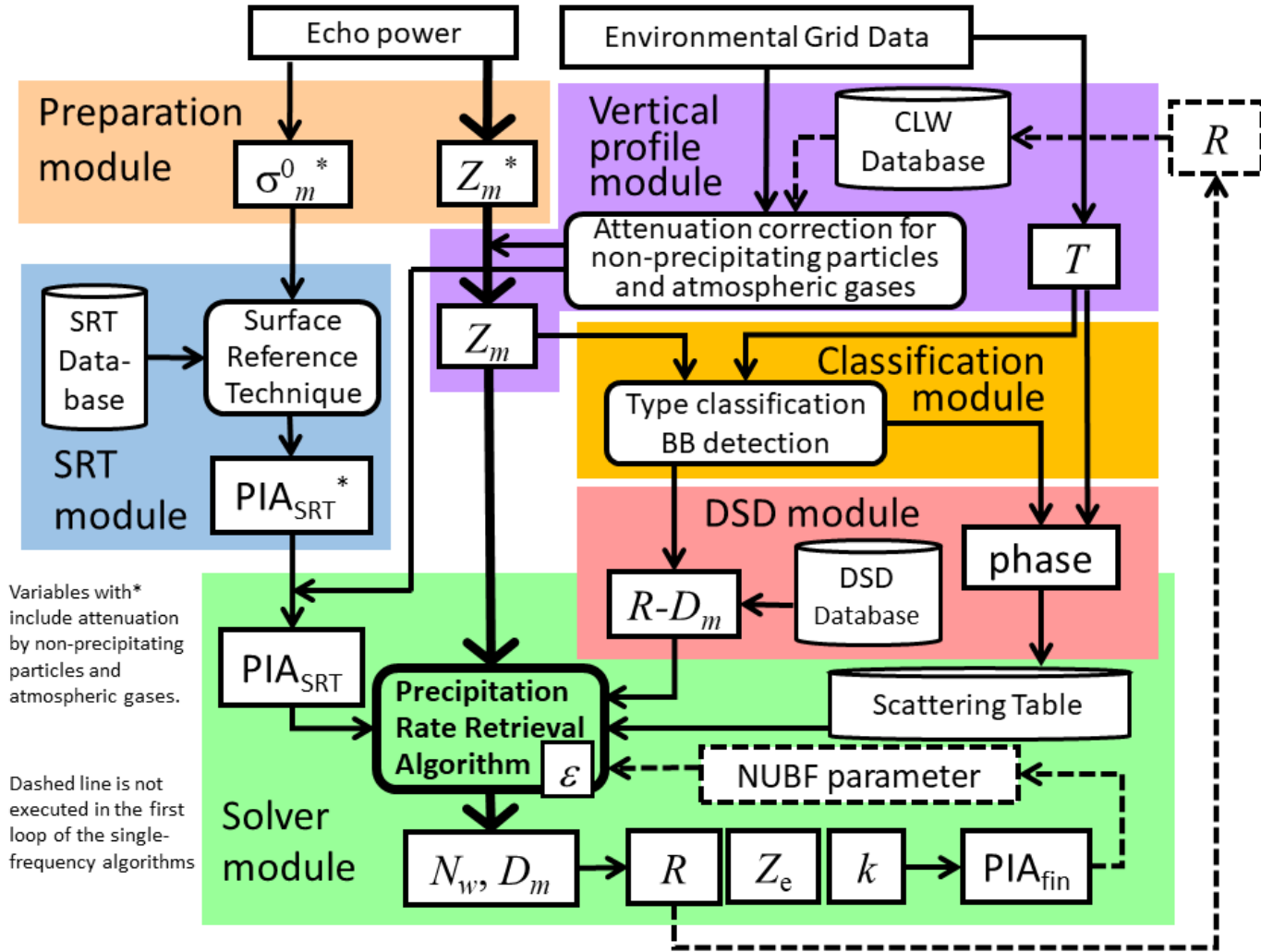
8. Performance

# DPR algorithms

- KuPR algorithm
  - KaPR algorithm
  - Dual-frequency algorithm
- } Single-frequency algorithms



# Modules of the DPR algorithms



Variables with\* include attenuation by non-precipitating particles and atmospheric gases.

Dashed line is not executed in the first loop of the single-frequency algorithms

Masaki et al. (2020)  
*IEEE Trans. Geosci. Rem. Sen.*  
 10.1109/TGRS.2020.3039978

Kubota et al. (2020)  
*J. Atmos. Ocean. Technol.*  
 10.1175/JTECH-D-20.0041.1

Awaka et al. (2021)  
*J. Meteorol. Soc. Japan*  
 10.2151/jmsj.2021-061

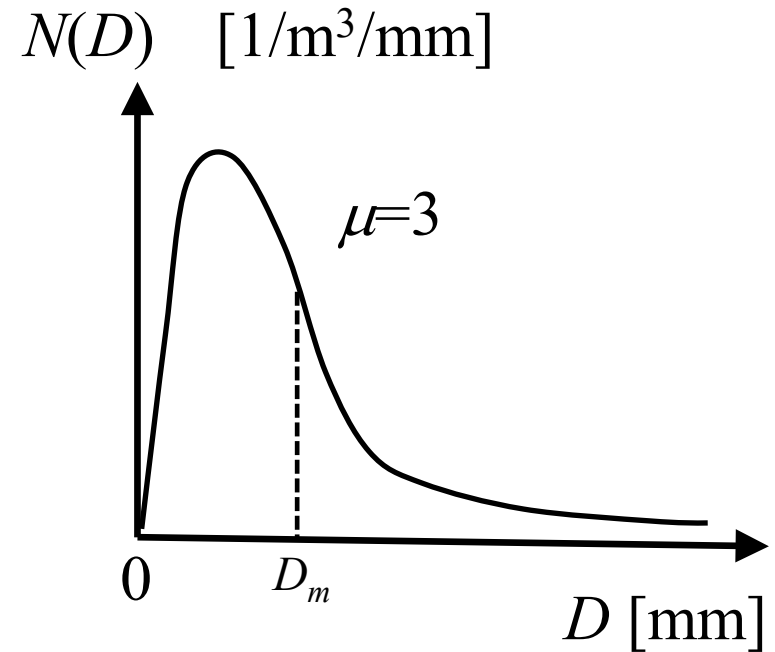
Meneghini et al. (2021)  
*J. Meteorol. Soc. Japan*  
 10.2151/jmsj.2021-010

Seto et al. (2021)  
*J. Meteorol. Soc. Japan*  
 10.2151/jmsj.2021-011

Seto (2024) **New!**  
*Advances in Weather Radar*  
 Volume 1, pp153-187.

## DSD model

$$N(D) = N_w f(D; D_m)$$



$$f(D; D_m) = \frac{6(\mu + 4)^{\mu+4}}{4^4 \Gamma(\mu + 4)} \left( \frac{D}{D_m} \right)^\mu \exp \left[ \frac{-(\mu + 4)D}{D_m} \right]$$

$$D_m = \frac{\int_{D=0}^{\infty} D^4 N(D) dD}{\int_{D=0}^{\infty} D^3 N(D) dD}$$

$D_m$ : mass-weighted mean diameter

# Rain rate

$$R = 0.6\pi \times 10^{-3} \int_{D=0}^{\infty} V(D)c(h)D^3 N(D)dD$$

$$= N_w \times 0.6\pi \times 10^{-3} \int_{D=0}^{\infty} V(D)c(h)D^3 f(D; D_m)dD$$



$$R = N_w f_R(D_m) c(h)$$

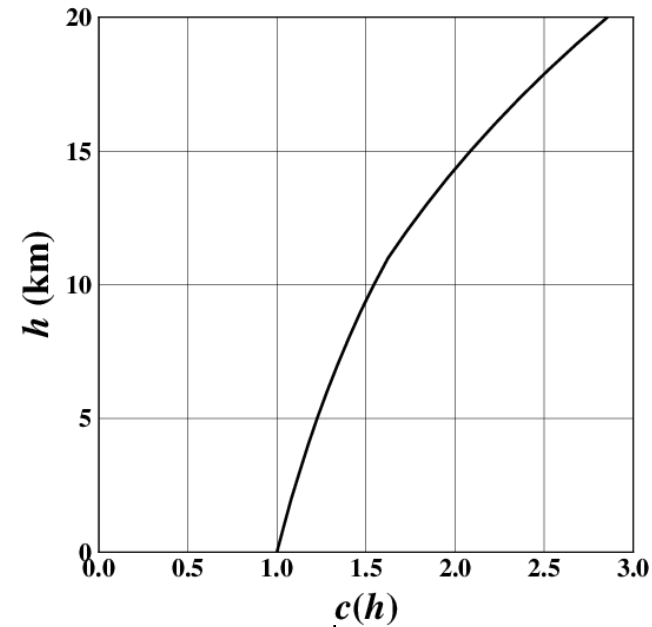
$$f_R(D_m) = 0.6\pi \times 10^{-3} \int_{D=0}^{\infty} V(D)D^3 f(D; D_m)dD$$

$$V(D) = 3.78D^{0.67}$$



$$f_R(D_m) = 0.6\pi \times 10^{-3} \times 3.78 \times \frac{6\Gamma(\mu + 4.67)}{4^4 (\mu + 4)^{0.67} \Gamma(\mu + 4)} D_m^{4.67}$$

$$= 0.1644 \times 10^{-3} \times D_m^{4.67}$$




# Radar Reflectivity Factor

$$Z_e = \frac{\lambda^4}{\pi^5 |K_w(\lambda)|^2} \int_{D=0}^{\infty} \sigma_b(D; \lambda, T) N(D) dD$$

$$= N_w \frac{\lambda^4}{\pi^5 |K_w(\lambda)|^2} \int_{D=0}^{\infty} \sigma_b(D; \lambda, T) f(D; D_m) dD$$

$\sigma_b$ : backscattering cross section

$$K_w(\lambda) \equiv \frac{n_{w0}(\lambda)^2 - 1}{n_{w0}(\lambda)^2 + 2}$$


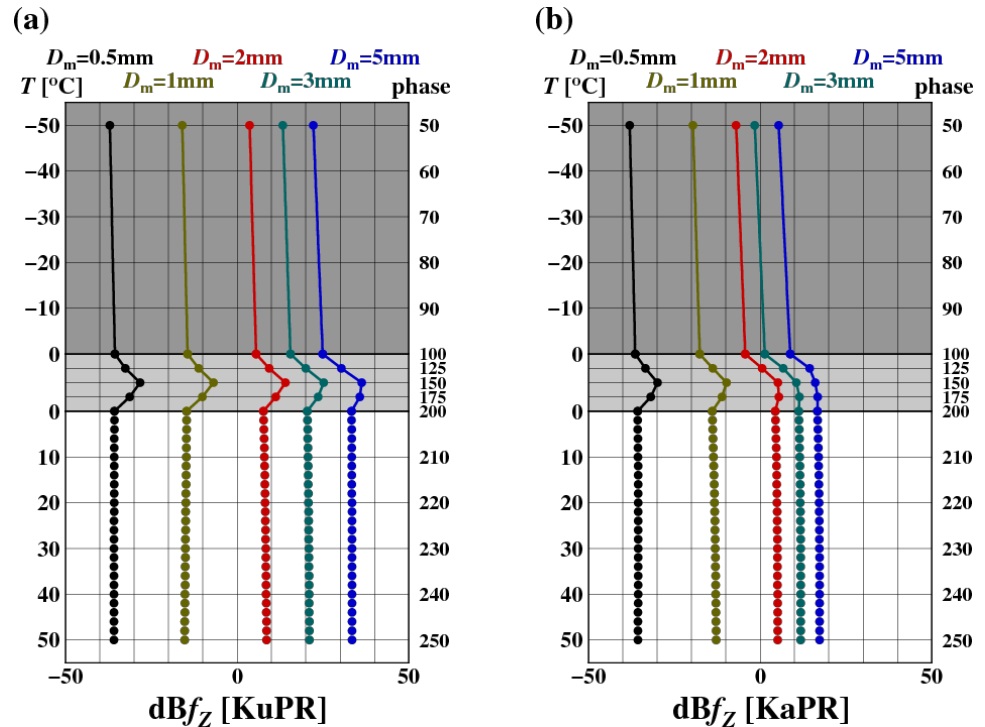
$$Z_e = N_w f_Z(D_m)$$

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# Specific Attenuation

$$k = \frac{0.01}{\ln 10} \int_{D=0}^{\infty} \sigma_e(D; \lambda, T) N(D) dD$$

$$= N_w \frac{0.01}{\ln 10} \int_{D=0}^{\infty} \sigma_e(D; \lambda, T) f(D; D_m) dD$$

$\sigma_e$ : extinction cross section



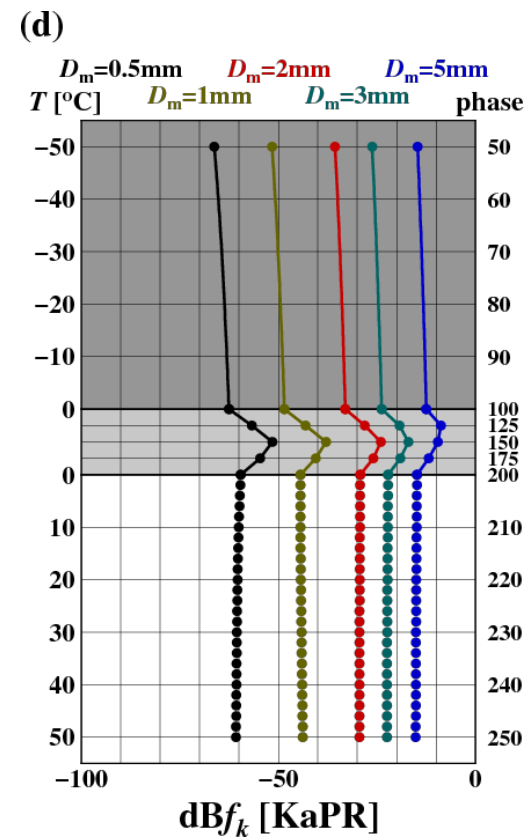
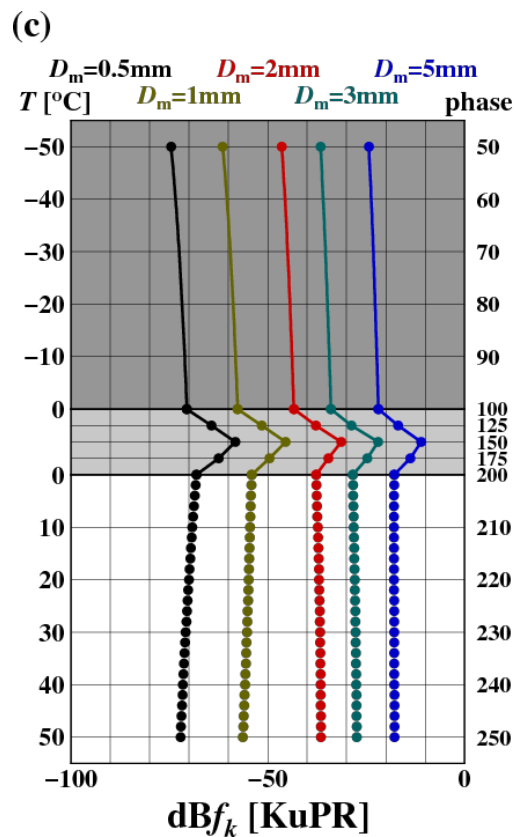
$$k = N_w f_k(D_m)$$

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# Measured radar reflectivity factor

$$\text{dBZ}_m(l) = \text{dBZ}_{e,i} - 2K_i L - 2k_i l$$

$$Z_m(l) = Z_{e,i} 10^{-0.2K_i L - 0.2k_i l}$$

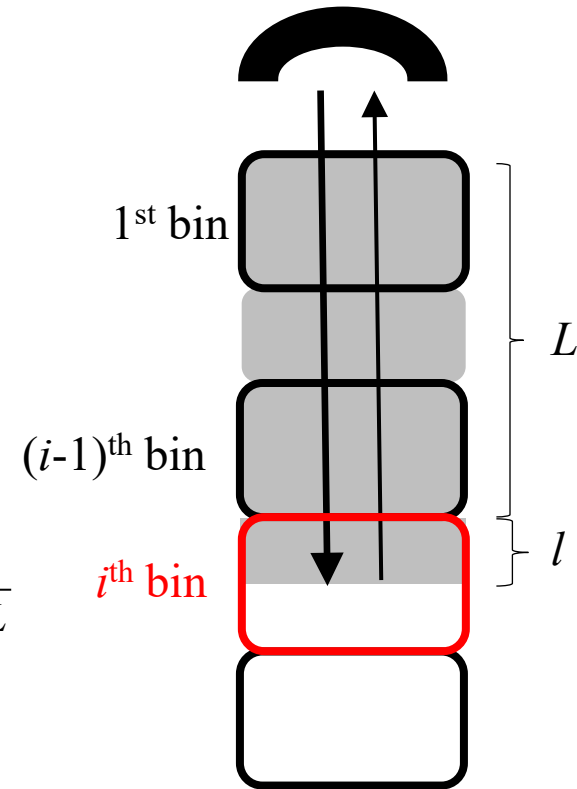
$$K_i \equiv \sum_{j=1}^{i-1} k_j$$

$$Z_{m,i} \equiv \frac{1}{L} \int_{l=0}^L Z_m(l) dl = \frac{1}{L} Z_{e,i} 10^{-0.2K_i L} \int_{l=0}^L 10^{-0.2k_i l} dl = Z_{e,i} 10^{-0.2K_i L} \frac{1 - 10^{-0.2k_i L}}{0.2(\ln 10)k_i L}$$

$$\text{dBZ}_{m,i} = \text{dBZ}_{e,i} - 2K_i L - \gamma k_i L$$

$\gamma$  satisfies the following eqn. and  $0 < \gamma < 1$ .

$$\frac{1 - 10^{-0.2k_i L}}{0.2(\ln 10)k_i L} = 10^{-0.1\gamma k_i L}$$



# Retrieval

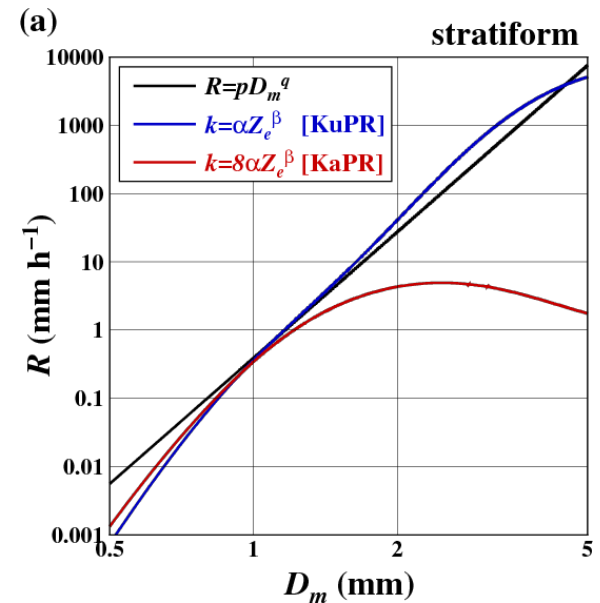
$$\text{dBZ}_m = \text{dBZ}_e - 2KL - \gamma kL$$

$$R = N_w f_R(D_m) c(h)$$

$$R = g(D_m)$$

$$Z_e = N_w f_z(D_m)$$

$$k = N_w f_k(D_m)$$



# R-Dm relation

$$Z \equiv \int_{D=0}^{\infty} D^6 N(D) dD$$



$$N(D) = N_w f(D; D_m)$$

$$Z = N_w \int_{D=0}^{\infty} D^6 f(D; D_m) dD$$

$$R = N_w f_R(D_m) c(h)$$



$$f(D; D_m) = \frac{6(\mu+4)^{\mu+4}}{4^4 \Gamma(\mu+4)} \left(\frac{D}{D_m}\right)^{\mu} \exp\left[-\frac{(\mu+4)D}{D_m}\right]$$

$$\mu = 3$$

$$Z = 0.034439 \times N_w D_m^7$$



$$V(D) = 3.78 D^{0.67}$$

$$R = 0.1644 \times 10^{-3} \times N_w D_m^{4.67}$$

$$Z = a R^b$$

$a = 298.84$  and  $b = 1.38$   
(Kozu et al. 2009)



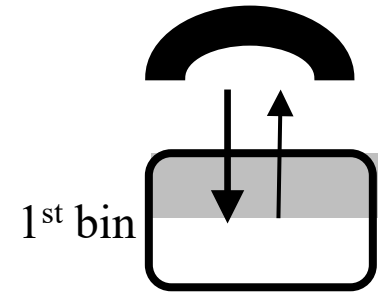
$$R = p D_m^q$$

$$p = \left( \frac{0.034439}{a \times 0.1644 \times 10^{-3}} \right)^{\frac{1}{b-1}} \quad q = \frac{2.33}{b-1}$$

# Retrieval ( $i=1$ )

At  $i=1$ ,  $K=0$ .

$$\text{dBZ}_m = \text{dBZ}_e - \gamma k L$$



$$R = N_w f_R(D_m) c(h)$$

$$R = g(D_m)$$

$$Z_e = N_w f_z(D_m)$$

$$N_w = \frac{g(D_m)}{f_R(D_m) c(h)}$$

$$k = N_w f_k(D_m)$$

$$\text{dBZ}_m = 10 \log_{10} \left[ \frac{g(D_m) f_z(D_m)}{f_R(D_m) c(h)} \right] - \gamma(D_m) \frac{g(D_m) f_k(D_m)}{f_R(D_m) c(h)} L$$

# Retrieval ( $i > 1$ )

At  $i > 1$ ,  $\text{dBZ}_f \equiv \text{dBZ}_m + 2KL$

$$\text{dBZ}_f = \text{dBZ}_e - \gamma kL$$

$$R = N_w f_R(D_m) c(h)$$

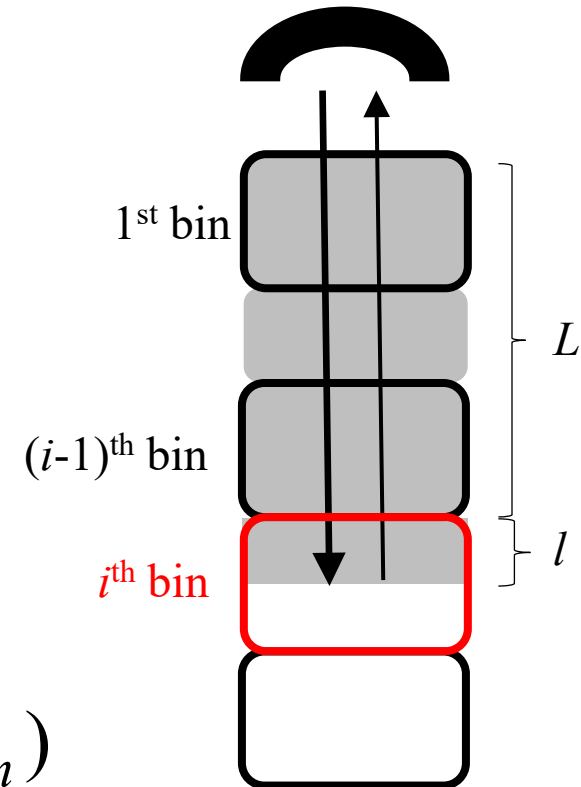
$$R = g(D_m)$$

$$Z_e = N_w f_z(D_m)$$

$$N_w = \frac{g(D_m)}{f_R(D_m) c(h)}$$

$$k = N_w f_k(D_m)$$

$$\text{dBZ}_f = 10 \log_{10} \left[ \frac{g(D_m) f_z(D_m)}{f_R(D_m) c(h)} \right] - \gamma(D_m) \frac{g(D_m) f_k(D_m)}{f_R(D_m) c(h)} L$$



# PIA (Path Integrated Attenuation)

$$\text{dBZ}_m = \text{dBZ}_e - 2KL - \gamma kL$$

$$R = N_w f_R(D_m) c(h)$$

$$Z_e = N_w f_z(D_m)$$

$$k = N_w f_k(D_m)$$

$$R = g(D_m)$$

SRT module

$$\text{PIA}_g \equiv \sum_{i=1}^n 2k_i L \longleftrightarrow \text{PIA}_{\text{SRT}} \equiv \text{dB}\sigma_e^0 - \text{dB}\sigma_m^0$$



# Modification of R-Dm relation

$$R = N_w f_R(D_m) c(h)$$

$$k = \varepsilon \alpha Z_e^\beta$$

$$Z_e = N_w f_z(D_m)$$

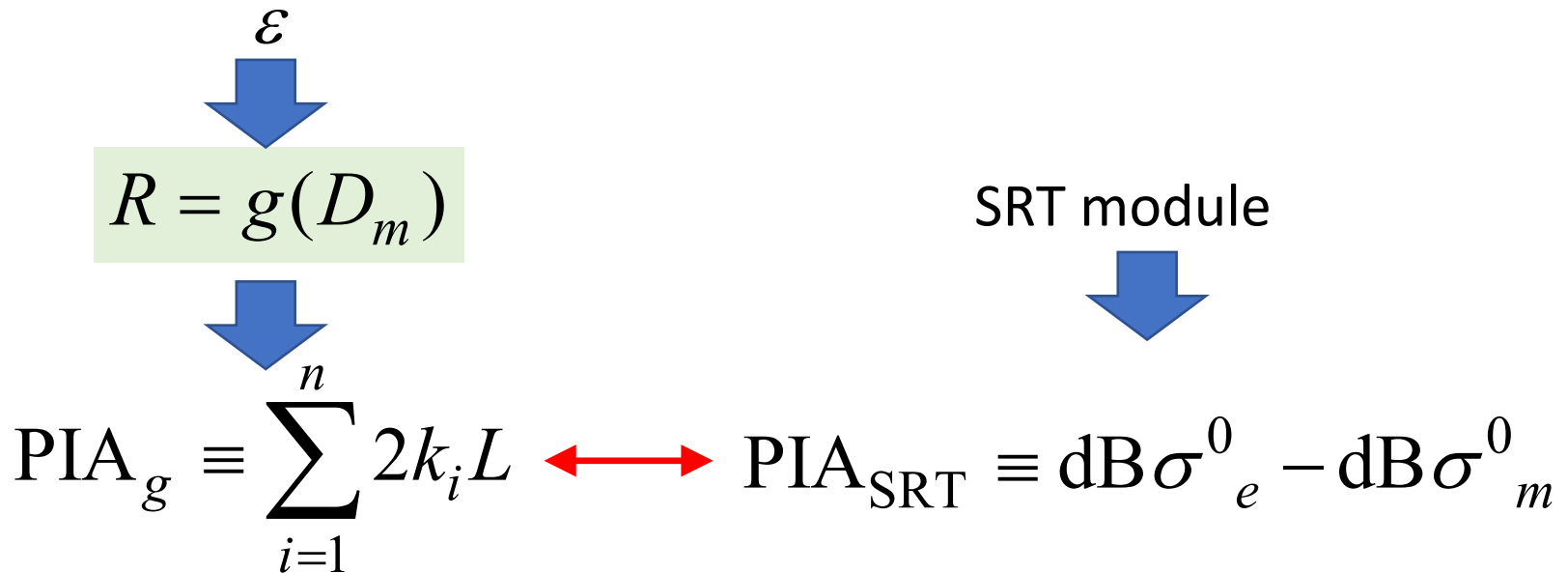
$$R = \left\{ \frac{\varepsilon \alpha [f_z(D_m)]^\beta}{f_k(D_m)} \right\}^{\frac{1}{1-\beta}} f_R(D_m) c(h)$$

$$k = N_w f_k(D_m)$$

$$R = g(D_m) = \varepsilon^r p D_m^q$$

$r = \frac{1}{1-\beta}$

# Maximum Likelihood method



$$p_1(x) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[-\frac{(x - \mu_x)^2}{2\sigma_x^2}\right] \quad x = \log_{10}\varepsilon$$

$$p_2(\text{PIA}_{\text{SRT}} | \text{PIA}_g) = \frac{1}{\sqrt{2\pi}\sigma_{\text{SRT}}} \exp\left[-\frac{(\text{PIA}_{\text{SRT}} - \text{PIA}_g)^2}{2\sigma_{\text{SRT}}^2}\right]$$

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