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## BACKGROUND AND MOTIVATION

- ❖ Single-scattering properties of hydrometeors are fundamental for physical precipitation (and cloud) remote sensing (retrieval).
- ❖ Many solid and melting hydrometeors have *complex and non-convex* shapes.
- ❖ The assumption of *simple, convex* shapes causes inconsistency between active (radar) and passive (radiometer) retrievals.
  - Olson et al (2016; <https://doi.org/10.1029/2015JD023907>)
- ❖ Generating the complex, non-convex hydrometeors and solving the associated EM scattering problems are computationally costly.
- ❖ Melting hydrometeors add the complication of **heterogeneous composition with high refractive contrast** between liquid and solid at lower microwave frequencies ( $\leq 35$  GHz).

## MIDAS IN A NUTSHELL

- ❖ MIDAS: MoM (Method of Moments) Integral-equation Decomposition for Arbitrary Scatterers
- ❖ MIDAS can run in one of two modes
  1. full MoM mode: **MIDAS-MOM**, no approximation
  2. Characteristic Basis Function Method mode: **MIDAS-CBFM**, with SVD approximation
- ❖ In the integral-equation formulation, MoM and DDA are equivalent
  - The volume elements, or voxels, in MoM are just like the dipoles in DDA
- ❖ They require the same criterion,  $|m|kd < 0.5$ , for accurate angular cross-sections where  $m$  is the refractive index,  $k$  the angular wave number, and  $d$  the dipole distance (voxel size).

## MIDAS-MOM

The volume integral equation (VIE)

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) + (k_0^2 + \nabla \cdot \nabla) \int_{\Omega} \chi(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') \mathbf{E}(\mathbf{r}') d\mathbf{r}'$$

$$\Rightarrow \mathbf{\Gamma} \mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r})$$

$$\mathbf{\Gamma} = \mathbf{I} + (k_0^2 + \nabla \cdot \nabla) \int_{\Omega} \chi(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') d\mathbf{r}': \text{operator}$$

$\Omega$ : the spatial domain (grid cells when discretized) occupied by the scatterer

$\chi(\mathbf{r}) = (\epsilon(\mathbf{r})/\epsilon_0) - 1$ : dielectric contrast between the scatterer and the free space  
After discretizing the *bounding box* of  $\Omega$  into a grid system of cubic cells, in which  $\Omega$  occupies  $N$  cells, we arrive at a linear system of size  $3N \times 3N$

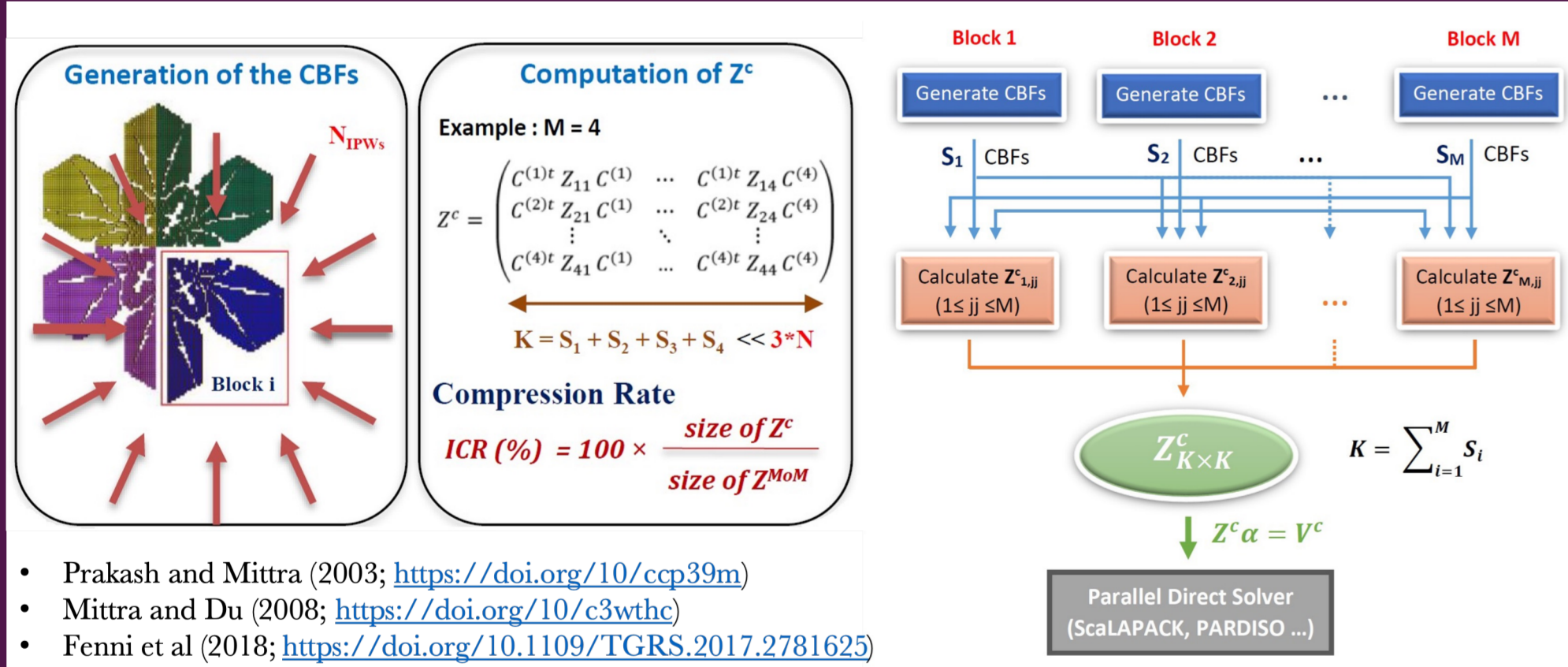
$$\sum_{n=1}^N \sum_{q=1}^3 Z_{pq}^{mn} E_q^n = E_{0,p}^m$$

## MIDAS-CBFM

In CBFM mode, MIDAS

- ❖ decomposes the container volume into several sub-volumes,
- ❖ formulates each sub-volume as a separate, local MoM problem with a set of incident waves,
- ❖ performs singular value decomposition (SVD) on the matrix of each local problem,
- ❖ applies a threshold to the singular values and constructs an approximate matrix by selecting only the components corresponding to those above the threshold,
- ❖ concatenates the local matrices, and
- ❖ solves the approximate problem.

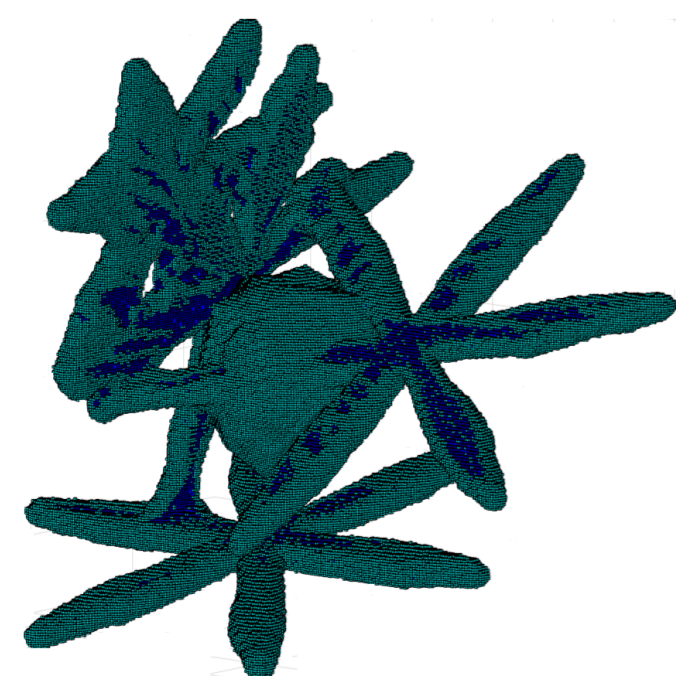
## MIDAS-CBFM ILLUSTRATION



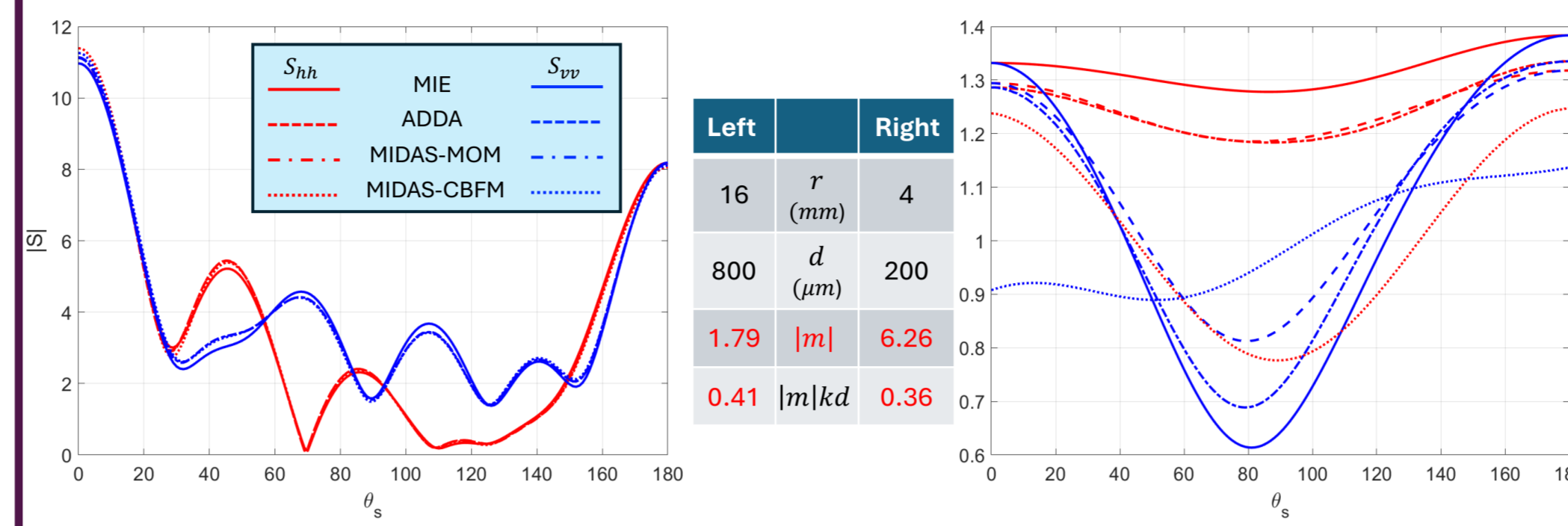
## CHALLENGE OF MELTING HYDROMETEORS

- ❖ Melting hydrometeors have a heterogeneous composition of ice and liquid water
- ❖ Starting around the Ka-band  $|m| > 4$  for liquid water, beyond the valid range of DDA
- ❖ Even when the DDA criterion,  $|m|kd < 0.5$ , is strictly satisfied, we find significant uncertainties in  $Q_e$ ,  $Q_s$ , and  $Q_b$
- ❖ Large contrast in dielectric dipole size,  $|m_w|/|m_i| \geq 2.5$ , at the liquid-solid interfacial boundaries

	13.5 GHz (Ku)	35 GHz (Ka)	94 GHz (W)
Water	6.26 + i 12.98	4.07 + i 2.37	2.94 + i 1.39
Ice	1.79 + i 3.61 $10^{-4}$	1.79 + i 9.13 $10^{-4}$	1.79 + i 2.41 $10^{-4}$
$ m_w / m_i $	3.87	2.63	1.82

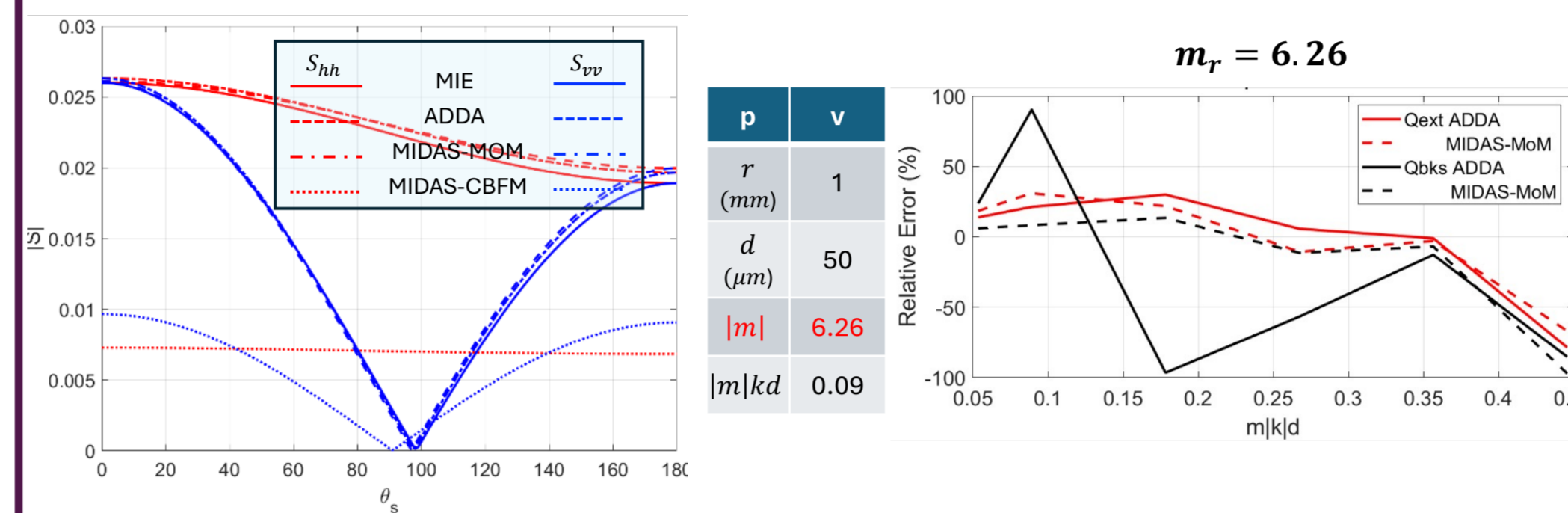


## HOMOGENEOUS LIQUID SPHERE



**Findings:**

1. The solution error is due mainly to **high** refractive index, not the contrast between them.
2. The  $|m|kd \lesssim 0.5$  criterion is valid for  $|m| = 1.79$ , but insufficient for larger  $|m|$ .

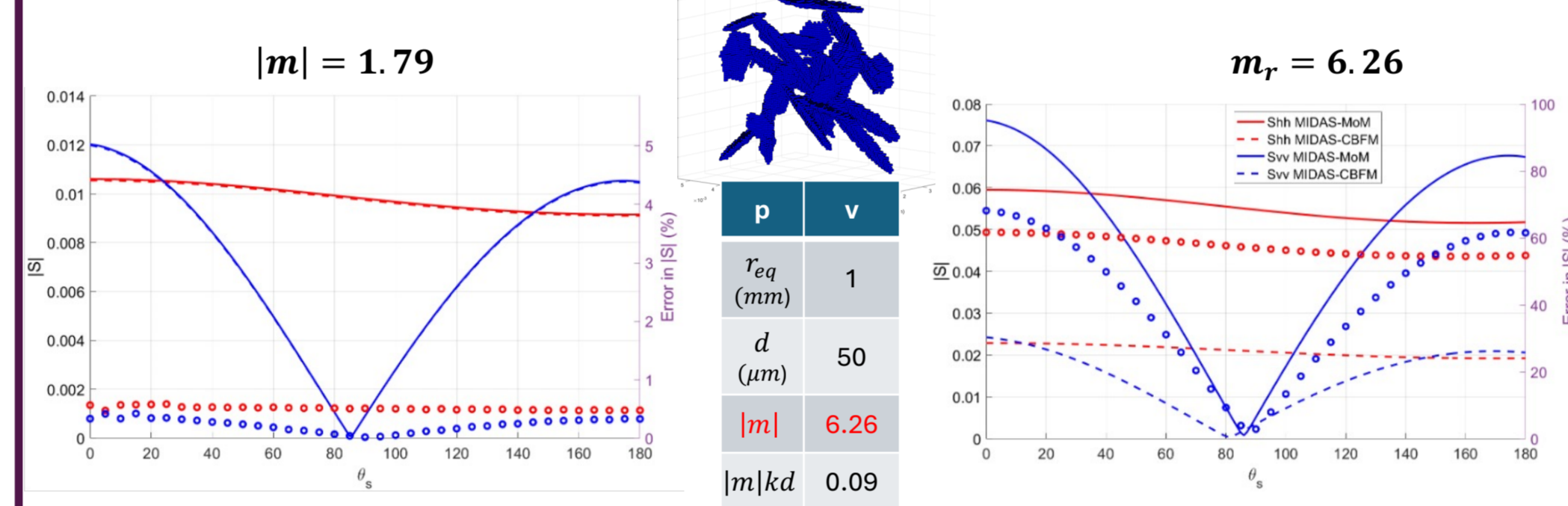


**Findings:**

3. Decreasing  $d$ , hence  $|m|kd$ , improves accuracies of ADDA and MIDAS-MoM for  $|m| = 6.26$
4. Higher resolution does not improve MIDAS-CBFM results.

## HOMOGENEOUS "LIQUID AGGREGATE"

### COMPARING MIDAS-CBFM TO -MOM (NO MIE)



**Finding:**

5. It is not the shape of the particle.

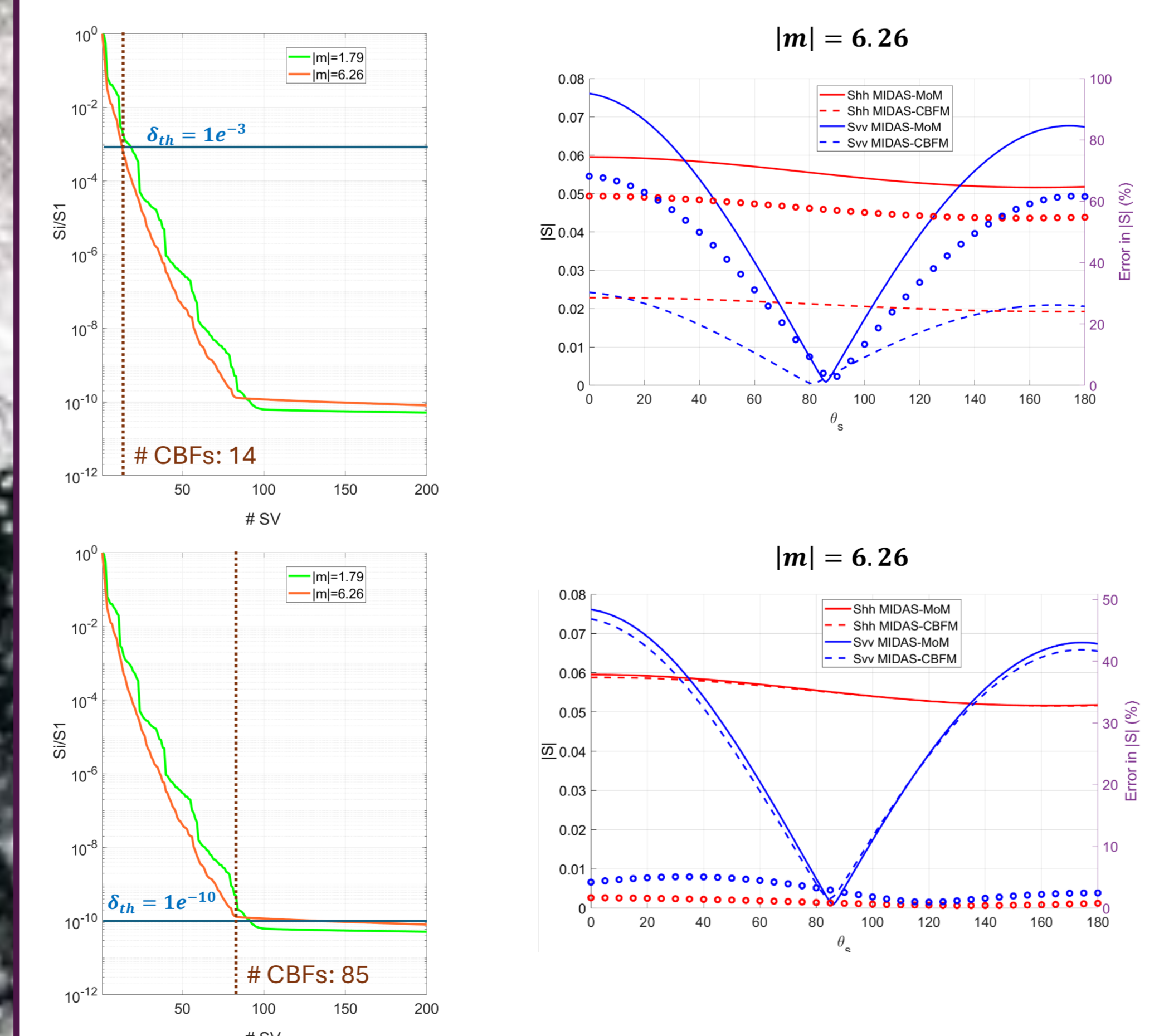
## RECAP

❖ Findings

- We initially suspected it was due to the high contrast between the refractive indices of liquid water and ice.
  - ❑ Comparing to the Mie solution of a liquid sphere shows that it is mainly due to the high refractive index of liquid water
  - ❑ The discretization resolution,  $d$ , improves ADDA and MOM solutions but not CBFM
- Results from a fictitious "liquid aggregate" indicate that particle shape has little or no impact.
- ❖ Since CBFM is an approximation to MOM with many tunable parameters, adjustment to some combination of these parameters may improve the approximation, e.g.,
  - the size (dimensions) of the CBFM blocks,
  - the number and type of the incident waves,
  - the quadrature used for integration, and
  - the threshold applied to the singular value decomposition (SVD).

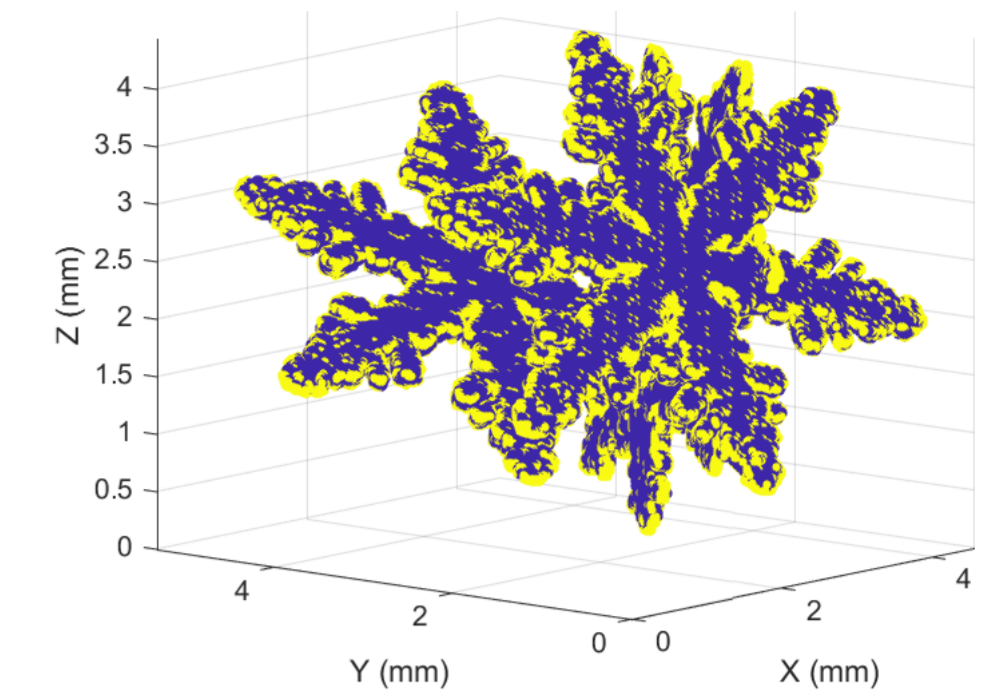
## HOMOGENEOUS LIQUID SPHERE

### THE ROLE OF SVD THRESHOLD



## BACKGROUND AND MOTIVATION

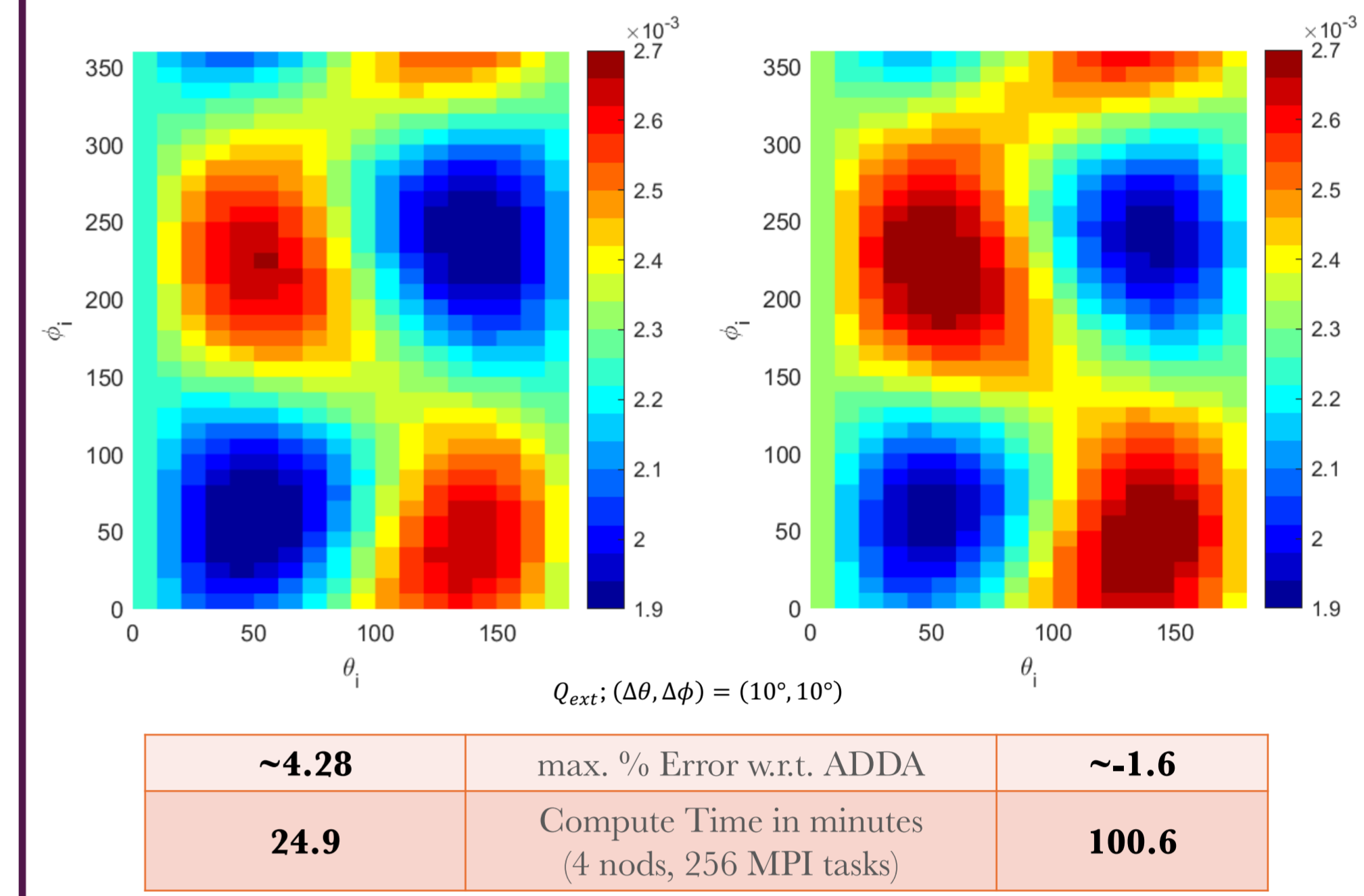
p	v
m	6.26
d (μm)	15
m kd	0.027
r_eq (mm)	0.834
D_max (mm)	5.32
LWF	3.285%



$$|m|kd \approx 0.027 (\approx 0.5/20)$$

- ❖ It takes ADDA ~4 minutes for each orientation and ~46 hours for 703 orientations!
- ❖ Even with significantly more CBFs, MIDAS has a large edge in time-to-solution!

$10^{-3}$	$\delta_{th}$	$10^{-8}$
adaptive	Quadrature	spherical design
190	$N_{iw}$	482



## CONCLUSIONS

- ❖ Realistic scatterer geometry and composition are paramount for reducing uncertainties in physical particulate-matter retrievals, e.g., aerosol, cloud, and precipitation.
- ❖ For higher  $|m| (\geq 4)$ , the accuracy of ADDA and MIDAS-MoM improves when a much finer discretization resolution is used, e.g.,  $|m|kd \lesssim 0.1$ , but not MIDAS-CBFM
  - Higher discretization resolutions lead to higher computation demands!
- ❖ The **SVD threshold used** for selecting CBFs at local blocks plays the most crucial role in improving MIDAS-CBFM accuracy (after resolution refinement)
  - The number of incident waves and the degree of quadrature play important supporting roles.
  - MIDAS-CBFM is still significantly more computationally competitive than DDA.
- ❖ MIDAS characterization effort will continue to address the issue of predetermining the optimal parameter combination.

## ACKNOWLEDGEMENT

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