Department of Meteorology[1]
Department of Mathematics and Statistics[2]

University of Reading

EPSRC
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Electromagnetic Wave Scattering by Complex Ice Crystals and Snowflakes

Karina McCusker[1,2] | Chris Westbrook[1] | Andrea Moiola[2]
karina.mccusker@pgr.reading.ac.uk

1. Introduction

- **Non-spherical ice crystals can be found in clouds below 0℃, and these can greatly impact the Earth's radiation budget.**
- **Accurate scattering models are required for radar and radiometer interpretations, to improve retrievals and increase climate modelling capabilities.**
- **The overall aim of this project is to develop an efficient and accurate method to approximate electromagnetic wave scattering by realistic ice particle shapes.**

2. Rayleigh-Gans Approximation (RGA)

- Computationally cheap method to approximate scattering properties.
- Assume total scattering from a particle can be calculated by dividing it into small volume elements and treating each element as a Rayleigh point scatterer.
- Neglect interactions between elements.

I used RGA to develop a scattering model in Matlab by:

- Calculating the particle's form factor, which is simply an integral:

$$f = \left| \frac{1}{v} \int_v \exp(2ki\mathbf{r} \cdot \hat{\mathbf{e}}_{\mathrm{i}})\, dv \right|^2$$

$v =$ particle volume
$k = \frac{2\pi}{\lambda} =$ wavenumber
$\mathbf{r} =$ particle radius
$\hat{\mathbf{e}}_{\mathrm{i}} =$ direction of incident wave

- Using f to calculate the radar cross-section (backscatter):

$$\sigma_b = constant \times mass^2 \times f$$

I tested my model using known analytical form factor equations for shapes such as spheres, spheroids and circular cylinders. I also derived form factors for a hexagonal prism (the simplest realistic ice crystal shape). I used these to calculate backscatter with the incident wave reaching the particle in 2 different directions, x and y (Fig. 1).

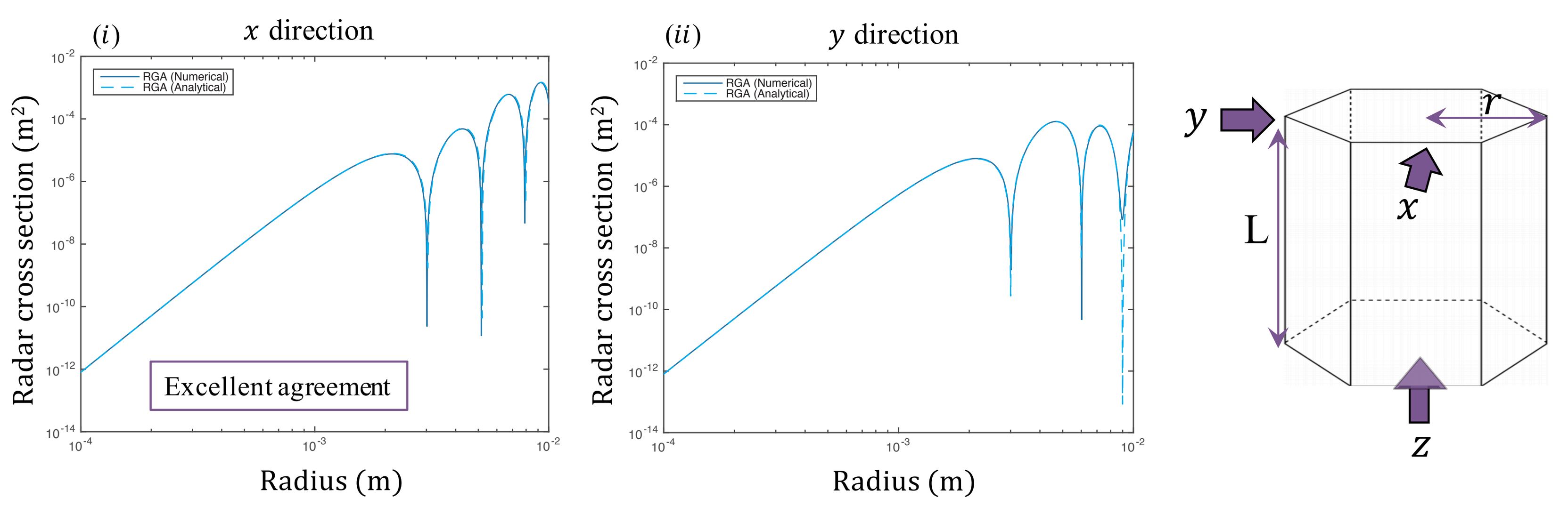

Figure 1: Radar cross section of hexagonal prisms with aspect ratio=1. The incident wave ($\lambda = 9mm$) reaches the particle at ***(i)*** a flat side; ***(ii)*** a pointy edge. The dark blue curves show the numerical solution and the dashed cyan curves show the derived analytical solution. For these calculations, the grid enclosing each particle is discretized into 80^3 volume elements.

Wave from x direction:

$$f = \left(\frac{L}{kv} \left(\frac{r}{u}(1 - \cos u) + r \sin u \right) \right)^2$$

Wave from y direction:

$$f = \left(\frac{uL}{k^3 rv} (\cos kr - \cos 2kr) \right)^2$$

$u = \sqrt{3}kr$

To examine the applicability of the RGA itself, I compared the RGA scattering model with the exact Mie solution to Maxwell's equations for spheres. Fig. 2 shows the radar cross-section of spheres of different radii plotted using various values of refractive index, m.

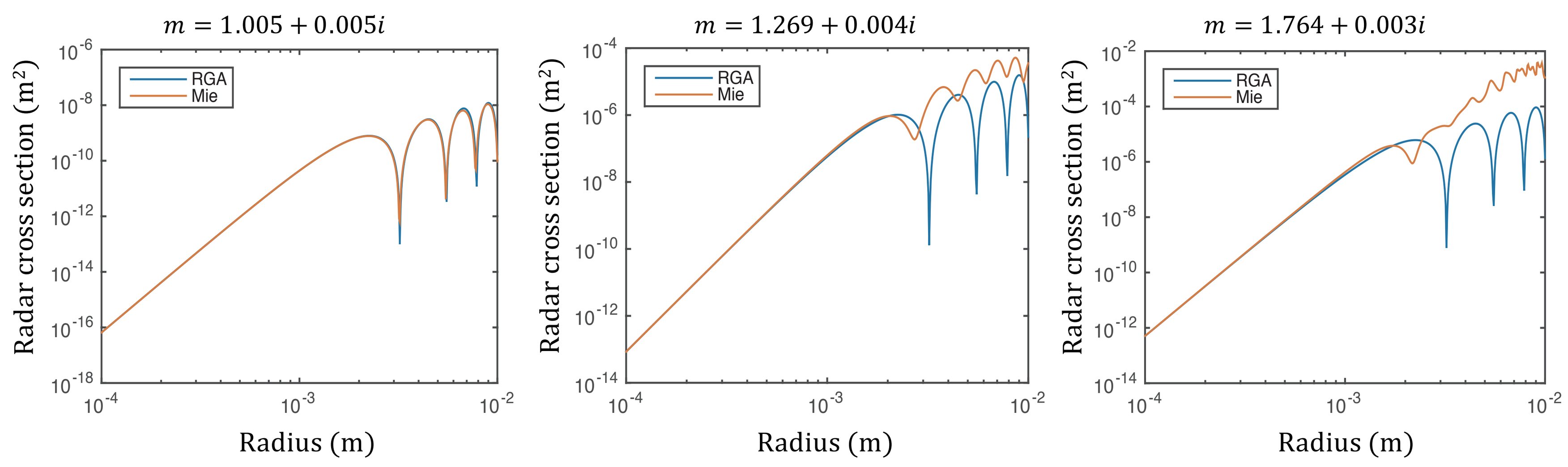

Figure 2: Radar cross-section of spherical particles using the Rayleigh-Gans scattering model (blue curves), and the Mie scattering model[a] of Christian Mätzler (orange curves), with $\lambda = 9mm$.

- For $m \approx 1$ (representative of large fluffy snowflakes) – RGA and Mie plots are similar for spheres of all radii.
- For $m \approx 1.7$ (representative of solid ice particles) – plots diverge for spheres of radii exceeding $\sim 1mm$.

- **RGA may be good for sparse geometries that have weaker interactions between volume elements.**
- **RGA is not accurate for dense particles greater than $\sim 1mm$ in radius, as interactions between volume elements are significant.**

3. Discrete Dipole Approximation (DDA)

Method to calculate scattering properties with *greater degree of accuracy*. However, a system of linear equations must be solved which makes DDA *more computationally expensive* than RGA.

- Shape is discretized into an array of polarizable points (dipoles).
- Dipole interactions with an applied electromagnetic field are calculated.
- Interactions with induced fields in other dipoles also taken into account.

I created my own DDA code and have used it to compare DDA and RGA.

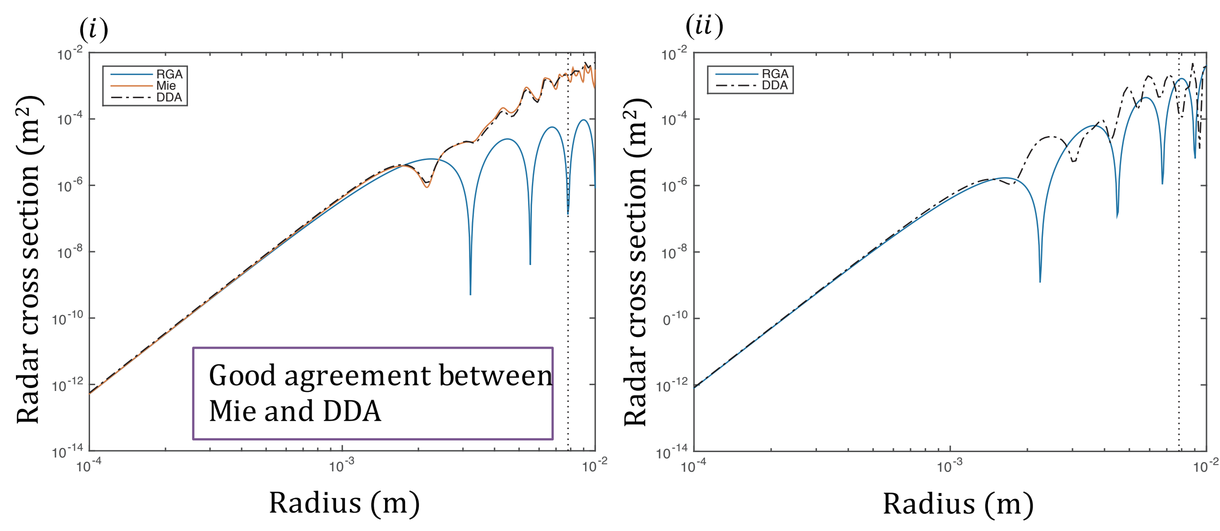

Figure 3: σ_b for
(i) Spheres
(ii) Hex. prisms (aspect ratio = 1)
Dotted line: DDA validity criterion[b] for no. of dipoles used is no longer satisfied.

I have also begun to examine the internal electric field of equal-volume spheres and hexagonal prisms.

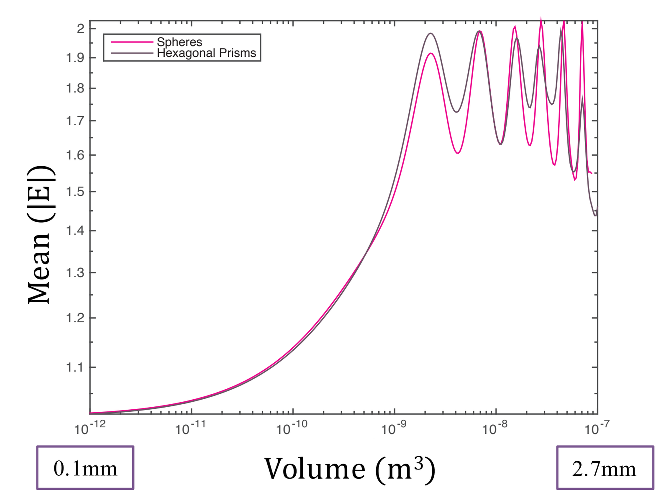

Figure 4: Mean of the magnitude of the internal electric field of spheres and hexagonal prisms of different volumes. Corresponding radii of spherical particles are labelled in boxes below the plot for reference.

- RGA: Internal field = Applied field = 1
- DDA: Internal field > 1

4. Next Steps

- Further examine the internal electric fields and compare solutions obtained using DDA to those obtained using RGA.
- Figure out the most important physics in DDA, the major weaknesses in RGA, and use this knowledge to develop a new fast, accurate scattering method somewhere in between RGA and DDA.

References

[a] Mätzler, C., MATLAB Functions for Mie Scattering and Absorption, Institut für Angewandte Physik, Research Report No. 2002-08, Bern, Switzerland, 2002.

[b] Draine, B. T., and P. Flatau, 1994: Discrete-dipole approximation for scattering calculations. *J. Opt. Soc. Amer.*, 11A, 1491–1499.

DDA in Action

If you would like to see the comparison of the electric field through slices of a hexagonal prism and sphere of equal-volume, visit q-r.to/bafrGi or scan the code: