1. INTRODUCTION
There are several factors that contribute to the uncertainties in raindrop size distributions (DSDs) retrieved from vertically pointing profiling radars. These factors include the temporal and spatial scales of the microphysical processes of the precipitation relative to the radar observations. For example, changes in the DSD due to microphysical processes within the spatiotemporal resolution of the radar observation will not be resolved by the radar. Other factors that contribute to the uncertainties of the retrieved DSDs include the instrument uncertainties of the radar observations, the mathematical models used to describe the DSD, and the numerical methods used to estimate the DSD. This study evaluates several numerical methods used to estimate the DSD using profiler observations collected at Darwin, Australia.

2. MATHEMATICAL DESCRIPTION OF DSD
Vertically pointing Doppler radar profilers sensitive to detecting raindrops observe the Doppler velocity of the raindrop which consists of the raindrop terminal fall speed augmented by the atmospheric air motion. Raindrops that are in a downdraft will have a measured downward Doppler velocity greater than their terminal fall speed. And raindrops that are in an updraft will fall slower than their terminal fall speed. And if the updraft is strong enough, small drops will actually be lifted to higher altitudes. Thus, in order to estimate the raindrop size distribution from Doppler velocity spectra, the vertical air motion must be estimated and used to shift the Doppler velocity spectra so that it becomes the terminal fall speed spectra.

While vertical air motion shifts the Doppler spectra away from the desired terminal fall speed spectra of hydrometeors, the finite beam width of the radar system causes the returned energy that should be assigned to a single spectral bin to be spread across several neighboring spectral bins. The returned energy is not lost, but is spread across neighboring spectral bins dependent on the radar beam width and the amount of atmospheric turbulence. Thus, the observed Doppler velocity spectrum has been broadened due to the finite beam width and the atmospheric turbulence.

The Doppler velocity spectrum from a vertically pointing profiler sensitive to the Rayleigh scattering from hydrometeors can be expressed mathematically as

\[ S_{\text{observed}}(v) = \frac{1}{\sqrt{2\pi\sigma_{\text{air}}}} \exp \left( -\frac{(v - \bar{v})^2}{2\sigma_{\text{air}}^2} \right) \ast S_{\text{hyd}}(v) \]  

(1)

where * is the symbol of the convolution operator, the function on the left of the convolution operator symbol represents the spectral broadening and shift due to the atmospheric air motion, and the function on the right of the convolution operator represents the raindrop size distribution in units of reflectivity factor and is a function of raindrop velocity.

3. NUMERICAL METHODS ESTIMATING THE DSD
Provided that the air motion, \( \bar{v} \), and the spectral broadening, \( \sigma_{\text{air}} \), are available from the profiler observations, there are two main classes of numerical methods used to estimate the DSD. The first class uses a deconvolution routine to correct for the spectral broadening. The spectrum is then shifted due to the air motion estimate and converted into a DSD estimate. The second class uses a convolution routine to broaden a modeled DSD spectrum. This second class iterates through multiple modeled DSD spectrums and repeats the convolution routine for each modeled DSD spectrum. The convolved modeled DSD spectrum is compared with the observed spectrum and the iteration is repeated until a goodness-of-fit score is minimized.

3.1 DSD Estimates using Deconvolution Routine
In this study, the observed Doppler velocity spectra were deconvolved using the FFT routine described in Lucas et al. (2004). One of the uncertainties with performing a deconvolution is that noise in the spectrum can be amplified such that the amplitude of the noise is comparable with the amplitude of real atmospheric signals. The Lucas et al. (2004) deconvolution method performs the deconvolution
using several different FFT coefficients and keeps the solution with the lowest amplified noise.

After the deconvolution of the original spectrum has been performed, the deconvolved spectrum is shifted by the air motion estimate. The resulting spectrum represents the reflectivity factor for each raindrop fall speed. After adjusting the raindrop terminal fall speed by the decrease in atmospheric density with increasing altitude, the raindrop diameter can be assigned to each velocity bin. The discrete raindrop size distribution is then estimated using

\[ N(D) = \frac{S(v_{\text{raindrop}})}{D_{\text{raindrop}}^\mu} \]  

where \( S(v_{\text{raindrop}}) \) is the reflectivity factor for each velocity, and \( v_{\text{raindrop}} \) is the atmospheric density adjusted terminal fall speed of the raindrop with diameter \( D \).

After \( N(D) \) is estimated from the deconvolved spectrum, then the raindrop size distribution can be estimated using several different mathematical descriptions of the DSD. In this abstract, the deconvolved spectrum is converted into DSD estimates using 5 different methods. These methods are listed in Table 1 and include the discrete DSD using equation (2), assuming a modified gamma functional form (Ulbrich 1983) with \( \mu \) being estimated (with two methods) and the special cases with \( \mu = 0 \) and \( \mu = 2.5 \).

3.2 DSD Estimates using Convolution Routine
The DSD estimation methods that utilize the convolution routine must iterate through several DSD estimates and compare the convolved DSD modeled spectrum with the observed spectrum until a goodness-of-fit score is minimized. This method requires that the functional form of the DSD be specified. And in this study, it is assumed that the raindrop size distribution has a modified gamma functional form. The DSD was estimated 4 different ways with \( \mu \) constrained to the values of 0 and 2.5 in two of the methods. The different methods are listed in Table 1.

4. OBSERVATIONS FROM DARWIN, AUSTRALIA
Two vertically pointing Doppler radar profilers have been deployed next to each other in Darwin, Australia. One profiler has an operating frequency of 50-MHz and observes the vertical air motion by detecting the Bragg scattering due to changes in the radio refractive index of turbulence. The second profiler has an operating frequency of 920-MHz and observes the Doppler motion of the hydrometeors by detecting the Rayleigh scattering due to backscattered power from the hydrometeors. While the two radars have different beamwidths and different pulse lengths that prohibit both radars from observing the same volume of the atmosphere, both radars are synchronized to have the same one-minute temporal sampling resolution.

The 50-MHz profiler observations provide an estimate of the vertical air motion and the spectrum broadening. These estimates are interpolated from the original 500 meter vertical resolution to the 100 meter vertical resolution of the 920-MHz profiler observations. The 50-MHz profiler derived estimates of vertical air motion and spectral broadening are used in all of the DSD estimates.

Figure 1a shows the time-altitude cross-section of reflectivity from one rain event that passed over the profiler site on 16 February 2003. The rain event has a leading convective core immediately followed by a stratiform rain regime. The horizontal line in Figure 1a indicates the altitude of the subsequent DSD analysis with the reflectivity at this altitude of 3.3 km shown in Figure 1b.

The DSD was estimated for each spectrum at the altitude of 3.3 km using the 9 different numerical methods listed in Table 1. The mass weighted mean diameter, \( D_m \), for each DSD estimate is shown in Figure 1c. There is scatter in the \( D_m \) estimates with the most scatter occurring during the convective period near 1300 UTC. Figure 1d shows the estimated rain rate derived from the estimated DSD. The rain rate varies from less than 1 mm/hr between the two stratiform rain regimes (near 1530 UTC) to over 200 mm/hr during the convective period.

For each DSD estimate, the mean mass-weighted diameters and rain rates from each of the 9 different numerical models were averaged together to form ensemble estimates of \( D_m \) and R. The deviations of the individual estimates from the ensemble mean represent the uncertainties of each numerical method. Using the minute estimates between 1400 and 1700 UTC, the average standard deviation of \( D_m \) and R from the ensemble mean at each minute were 0.1 mm and 0.23 mm/hr. The rain rate was less than 8 mm/hr during this 3 hour period.

5. CONCLUDING REMARKS
Using 50- and 920-MHz profiler observations from Darwin, Australia, the uncertainties of estimating the raindrop size distribution from several numerical methods were evaluated. The 50-MHz profiler observations were used to estimate the vertical air motion and the spectral broadening due to turbulent motions. These air motion and broadening estimates were used along with the 920-MHz profiler Doppler velocity spectra to estimate the raindrop size distribution using 9 different numerical methods. These separate numerical methods were combined to form an ensemble estimate that represents the mean DSD parameters with the deviation of the individual methods from this ensemble mean providing an estimate of model uncertainties. During stratiform rain when the rain rate was less than 8 mm/hr, it was estimated that the standard deviation of the mass...
weighted mean diameter and rain rate were 0.1 mm and 0.23 mm/hr, respectively.

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6. REFERENCES

Table 1. Description of Numerical Methods used to Estimate the DSD

<table>
<thead>
<tr>
<th>Model #</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Deconvolution, Discrete DSD, equation (2)</td>
</tr>
<tr>
<td>2</td>
<td>Deconvolution, exponential function, μ=0, best fit to spectrum in D^6 space</td>
</tr>
<tr>
<td>3</td>
<td>Deconvolution, gamma function, μ=2.5, best fit to spectrum in D^6 space</td>
</tr>
<tr>
<td>4</td>
<td>Deconvolution, gamma function, μ=best fit, best fit to spectrum in D^6 space</td>
</tr>
<tr>
<td>5</td>
<td>Deconvolution, gamma function, μ=best fit, best fit to 0th, 1st, and 2nd moments</td>
</tr>
<tr>
<td>6</td>
<td>Convolution, exponential function, μ=0, best fit to spectrum in D^6 space</td>
</tr>
<tr>
<td>7</td>
<td>Convolution, gamma function, μ=2.5, best fit to spectrum in D^6 space</td>
</tr>
<tr>
<td>8</td>
<td>Convolution, gamma function, μ=best fit, best fit to spectrum in log10(D^6) space</td>
</tr>
<tr>
<td>9</td>
<td>Convolution, gamma function, μ=best fit, best fit to spectrum in D^6 space</td>
</tr>
</tbody>
</table>

Figure 1. 920-MHz profiler observations on 16 February 2003 above Darwin, Australia. (a) Time-altitude cross-section of reflectivity, (b) reflectivity at 3.3 km, (c) mass weighted mean diameter and (d) rain rate at 3.3 km estimated from 9 different numerical methods listed in Table 1.