Faraday Rotation Angle of PALSAR Data in Sendai Area

TOHOKU UNIVERSITY

Jiong CHEN Motoyuki SATO

2010/03

Content

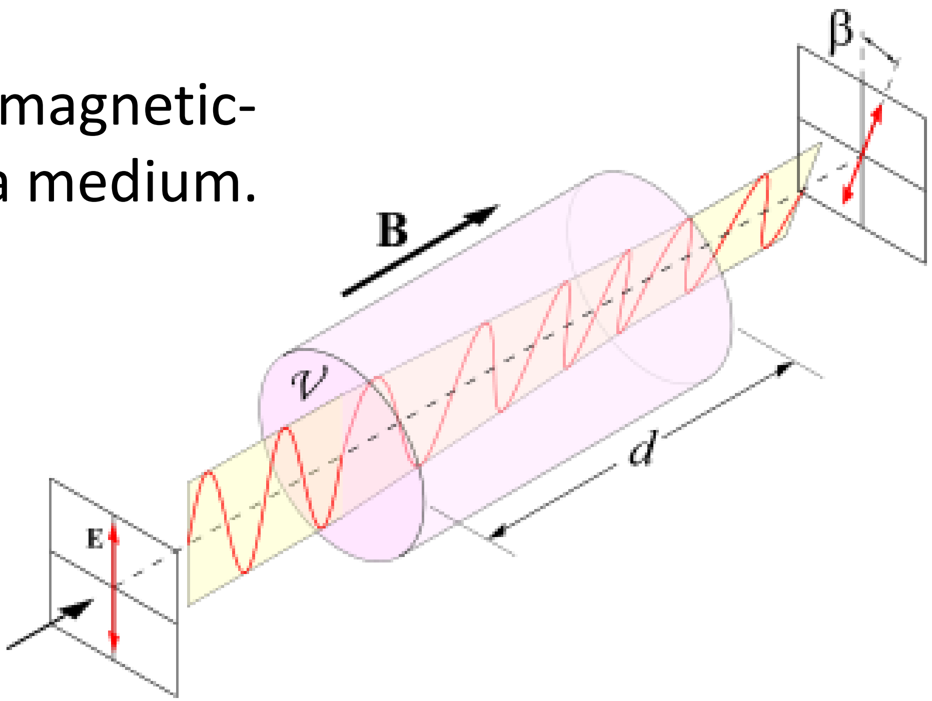
- Faraday Rotation Effect
- Estimation of Faraday Rotation
- A Robust Estimate and Discussion
- Conclusion

Faraday Rotation Effect

- Definition

  interaction between electromagnetic-wave and magnetic field in a medium.

$$\beta = \nu B d$$



β : Angle of rotation /radius

B : Magnetic flux density in the direction of propagation /tesla

d : Length of the path /meters

ν : Verdet constant for the material. Empirical constant varies with wavelength /radians · tesla^{-1} · meter^{-1}

Faraday Rotation Effect

- For space-borne polarimetric SAR

$$\Omega = \frac{K}{f^2}\int_0^h NB\cos\psi \sec\theta_0 dh$$

Ω: Angle of rotation /radius

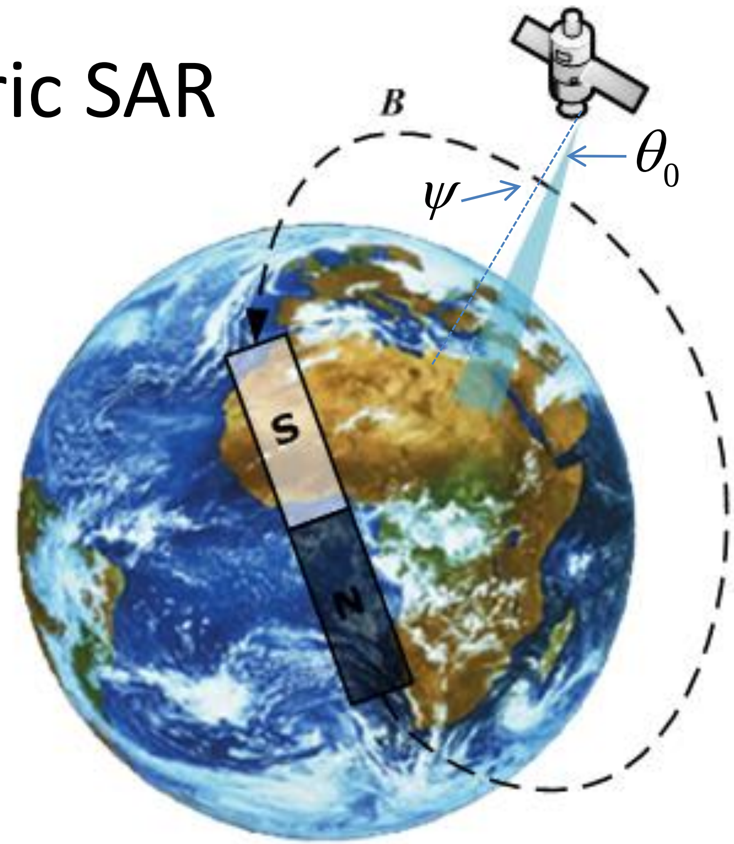
K: Constant value, 2.365×10^4

N: Electron Density

B: Magnetic flux density in the direction of propagation /tesla

h: Height of SAR platform /m

f: Frequency of wave /Hz



Faraday Rotation Effect

- Mathematical representation

$$Z = RFSFT + N$$

$$= \begin{bmatrix} 1 & \delta_1 \\ \delta_2 & f_1 \end{bmatrix} \begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix} \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & \delta_3 \\ \delta_4 & f_2 \end{bmatrix} + \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}$$

Nonreciprocal

O : Observed scattering matrix

S : Scattering matrix

F : One-way Faraday rotation matrix

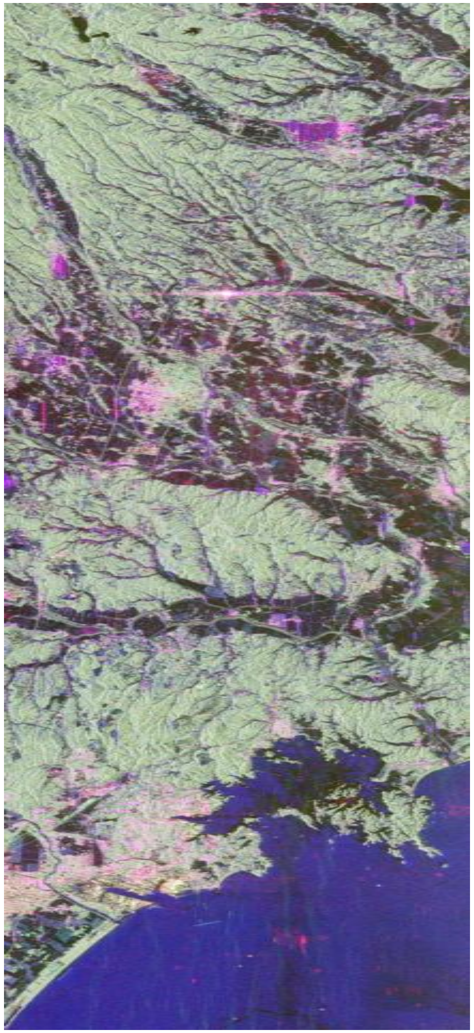
N : Noise matrix

R, T : Receive and transmit distortion matrix

Estimation of Faraday Rotation

- Scene: Sendai area

Parameter	Value
Inclination	98.16 degree
Carrier frequency	1.27GHz
Altitude	691.65km (above equator)
Scene center	38.5N, 141.0E
Incident angle	25.588 degree
Scanning time	20090604 12:54:33.292 (UT)
Direction	Ascendant
Polarimetry	HH/HV/VH/VV



Estimation of Faraday Rotation

- Approximation of $\bar{B}$ at 400km:

$$\Omega = KN_f \overline{B \cos\psi \sec\theta_0} \,/\, f^2$$

N_f : total electron content (TEC) per square meter

http://omniweb.gsfc.nasa.gov/vitmo/iri_vitmo.html

$$N_f \approx 8.0475e+16$$

$\bar{B}$: magnetic flux density at the height of 400km

http://wdc.kugi.kyoto-u.ac.jp/igrf/point/index-j.html

$$\Omega = -1.1317°$$

Estimation of Faraday Rotation

- Wright's Approximation:

$$\Omega = 0.299 \times \frac{\text{TECU}}{f_{GHz}^{2}} \times g\left(\theta,\phi\right)$$

Assume the earth magnetic field is generated by a magnetic dipole

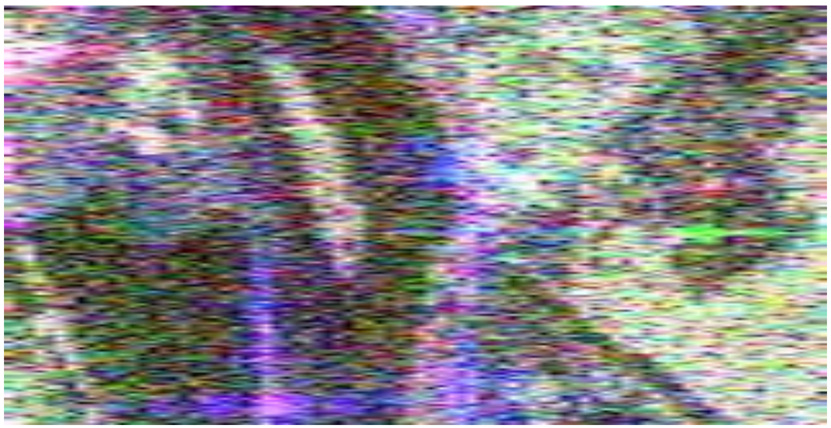
$$g\left(\theta,\phi\right) = 2\left(\sin\theta_m \sin\cos\left(\phi_m - \phi\right) + \cos\theta_m \cos\theta\right)$$

$$\pm \tan\theta_0 \left(\sin\theta_m \sin\lambda \sin\left(\phi_0 - \phi_m\right) \pm \cos\theta_m \cos\lambda\right)$$

$$\Omega = -2.0014^\circ$$

Estimation of Faraday Rotation

- Using reciprocal target



Target1: Trihedral

$$\begin{bmatrix} Z'_{HH} & Z'_{HV} \\ Z'_{VH} & Z'_{VV} \end{bmatrix} = \begin{bmatrix} 1 & \delta_1 \\ \delta_2 & f_1 \end{bmatrix}^{-1} \begin{bmatrix} Z_{HH} & Z_{HV} \\ Z_{HV} & Z_{VV} \end{bmatrix} \begin{bmatrix} 1 & \delta_3 \\ \delta_4 & f_2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 4.0695 + j1.3229 & 0.1196 + j0.0700 \\ -0.1473 - j0.1717 & 3.6275 + j1.6351 \end{bmatrix}$$

$$\begin{bmatrix} S_{LL} & S_{LR} \\ S_{RL} & S_{RR} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \begin{bmatrix} Z'_{HH} & Z'_{HV} \\ Z'_{VH} & Z'_{VV} \end{bmatrix} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2719 - j0.1699 & -1.3456 + j3.9694 \\ -1.6125 + j3.7277 & -0.1701 + j0.1422 \end{bmatrix}$$

$$\Omega = -\frac{1}{4} Arg\left(\left\langle S_{LR} S^{*}_{RL} \right\rangle \right) = -1.1665^{\circ}$$

Estimation of Faraday Rotation

- Summary

400km *B*	Wright's Appr.	Reciprocal
-1.1317°	-2.0014°	-1.1665°
$\bar{B}$ at 400km	Magnetic Dipole	Reciprocal target
Based on geophysical model		Based on the data with system distortion removed
Inaccuracy of geophysical parameters, time-varying geophysical conditions		Using only one target, not a robust estimate

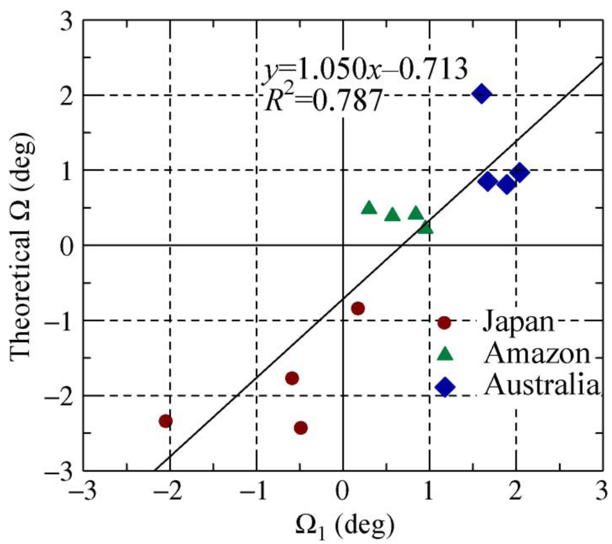
Estimation of Faraday Rotation

- Summary

400km *B*	Wright's Appr.	Reciprocal
-1.1317°	-2.0014°	-1.1665°

Sunspot number: 17 (Spotless day) → $N_f \approx 8.0475e+16$

http://spaceweather.com/archive.php?view=1&day=04&month=06&year=2009



Faraday rotation angle estimated by other researchers for reference

H. Kimura, "Calibration of polarimetric PALSAR imagery affected by Faraday rotation using polarization orientation," IEEE Trans. Geosci. Remote Sens., vol. 47, no. 12, pp. 3943–3950, Dec. 2009.

A Robust Estimate and Discussion

- Reciprocal target after Faraday rotation in circular basis

$$S'_{LR} = \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix} \begin{bmatrix} S_{HH} & S_{HV} \\ S_{HV} & S_{VV} \end{bmatrix} \begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_{HH} - S_{VV} + 2jS_{HV} & j\left(S_{HH} + S_{VV}\right)e^{-j2\Omega} \\ j\left(S_{HH} + S_{VV}\right)e^{j2\Omega} & -S_{HH} + S_{VV} + 2jS_{HV} \end{bmatrix}$$

$$\Omega = -\frac{1}{4} Arg\left(\left\langle S_{LR} S^*_{RL} \right\rangle\right)$$

Trihedral – good target for estimation

$$S_{HV} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow S'_{LR} = \begin{bmatrix} 0 & 2e^{-j2\Omega} \\ 2e^{j2\Omega} & 0 \end{bmatrix}$$

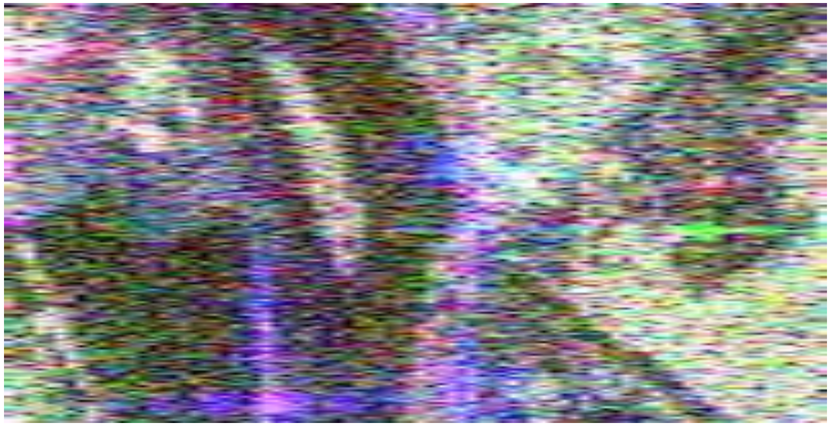
Dihedral – inapplicable for estimation

$$S_{HV} = \begin{bmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{bmatrix} \rightarrow$$

$$S'_{LR} = \begin{bmatrix} e^{j2\phi} & 0 \times e^{-j2\Omega} \\ 0 \times e^{j2\Omega} & e^{-j2\phi} \end{bmatrix}$$

A Robust Estimate and Discussion

- Dihedral – inapplicable for validation



Target2:

$$\begin{bmatrix} Z'_{HH} & Z'_{HV} \\ Z'_{VH} & Z'_{VV} \end{bmatrix} = \begin{bmatrix} 1 & \delta_1 \\ \delta_2 & f_1 \end{bmatrix}^{-1} \begin{bmatrix} Z_{HH} & Z_{HV} \\ Z_{HV} & Z_{VV} \end{bmatrix} \begin{bmatrix} 1 & \delta_3 \\ \delta_4 & f_2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0.2472 - j0.3428 & 11.4004 + j2.1968 \\ 11.7636 + j1.8664 & -0.2523 - j0.4301 \end{bmatrix}$$

$$\begin{bmatrix} S_{LL} & S_{LR} \\ S_{RL} & S_{RR} \end{bmatrix} = \begin{bmatrix} -1.7819 + j11.6256 & 0.2049 + j0.1627 \\ 0.5681 - j0.1677 & -2.2813 + j11.5384 \end{bmatrix}$$

$$\Omega = -\frac{1}{4} Arg\left(\left\langle S_{LR} S_{RL}^{*} \right\rangle\right) = 13.7253^\circ$$

A Robust Estimate and Discussion

- Similarity Parameter

Objective : Select trihedral (odd-bounce)-like target to obtain the distribution, and discard dihedrals.

$$S = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \to \vec{k} = \begin{bmatrix} S_{HH} & S_{VV} & S_{HV} & S_{VH} \end{bmatrix}$$

$$r(S_1, S_2) = \left| \left(\vec{k}_1^* \right)^t \vec{k}_2 \right| / \left(\left\| \vec{k}_1 \right\|_2^2 \left\| \vec{k}_2 \right\|_2^2 \right)$$

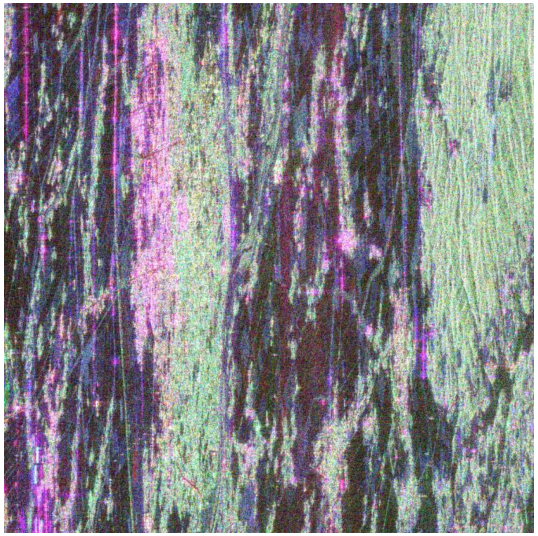
$$r\left(S, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) = 0.5 \left| S_{HH} + S_{VV} \right|^2 / \left(\left| S_{HH} \right|^2 + \left| S_{VV} \right|^2 + \left| S_{HV} \right|^2 + \left| S_{VH} \right|^2 \right)$$

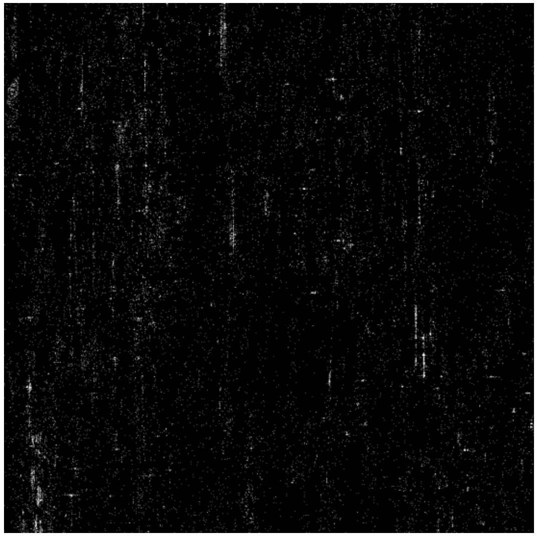
$$r\left(S, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right) = 0.5 \left| S_{HH} - S_{VV} \right|^2 / \left(\left| S_{HH} \right|^2 + \left| S_{VV} \right|^2 + \left| S_{HV} \right|^2 + \left| S_{VH} \right|^2 \right)$$

A Robust Estimate and Discussion

- $r\left(S, diag\left(1,1\right)\right) > 0.90 \; and \; r\left(S, diag\left(1,-1\right)\right) < 0.10$

<u>Original area</u>

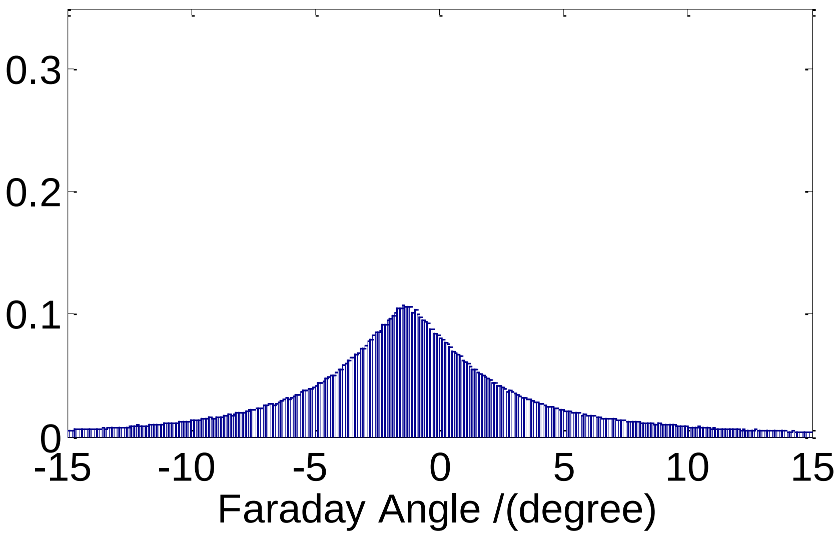




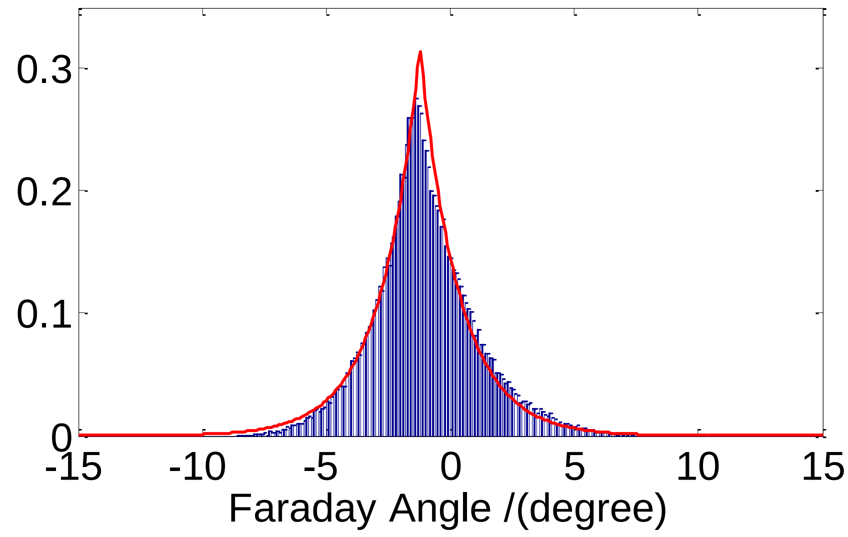
<u>Selection area</u>

	Original Area	Selected Area
Total Pixels	1000×1000	60160
Abs(FR)>10°	186862	0
Percentage	18.69%	0%

A Robust Estimate and Discussion



Distribution of Faraday Angle : Entire area



Distribution of Faraday Angle : Selected area

The distribution of Faraday rotation angle can be approximated by Laplace distribution

Laplace distribution : $$f\left(x \middle| \mu, b\right) = \frac{1}{2b} \exp\left(-\frac{|x-\mu|}{b}\right)$$

Maximum likelihood estimation (MLE): $\mu = -1.22 = FR$

Conclusion

- The Faraday rotation in Sendai area

400km *B*	Wright's	Reciprocal	Robust
-1.1317°	-2.0014°	-1.1665°	-1.2200°
Geophysical	Geophysical	Data-based	Data-based

- The robust estimate
  - Select trihedral targets; discard dihedral targets.
  - The MLE of the Laplace distribution
- Geophysical method *vs* Data-based method
  - Can validate the accuracy of calibration

Reference

[1] S. H. Bickel and R. H. T. Bates, “Effects of magneto-ionic propagation on the polarization scattering matrix,” Proc. IRE, vol. 53, pp. 1089–1091, 1965.

[2] A. Freeman, “Calibration of linearly polarized polarimetric SAR data subject to Faraday rotation,” IEEE Trans. Geosci. Remote Sens., vol. 42, no. 8, pp. 1617–1624, Aug. 2004.

[3] H. Kimura, “Calibration of polarimetric PALSAR imagery affected by Faraday rotation using polarization orientation,” IEEE Trans. Geosci. Remote Sens., vol. 47, no. 12, pp. 3943–3950, Dec. 2009.

[4] Yang J., Peng Y. N., Lin S. M., “Similarity between Two Scattering Matrices”, Electron. Letters, no. 3, pp. 193-194, 2001.

[5] Abramowitz, M. and Stegun, I. A. (Eds.). ”Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables”, 9th printing. New York: Dover, 1972.