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A New Model-Based Scattering Power Decomposition for Polarimetric SAR and Its Application in Analyzing Post-Tsunami Effects

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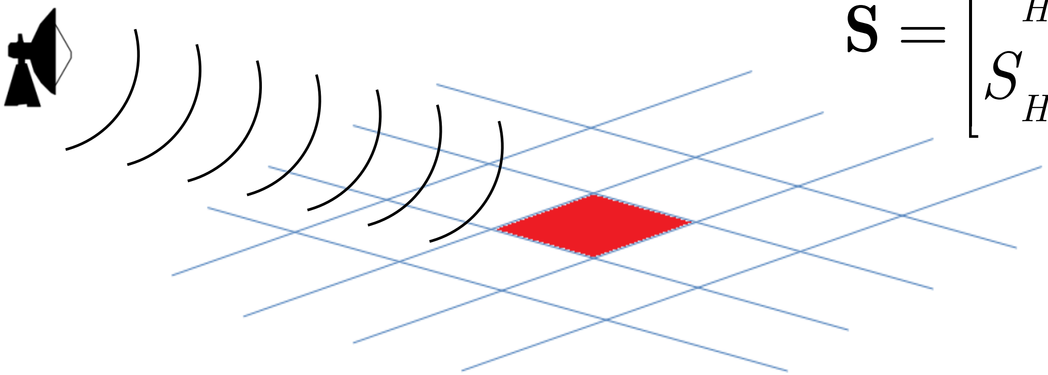
Background (1/5)

- POLSAR data



$$\mathbf{S} = \begin{bmatrix} S_{HH} & S_{VH} \\ S_{HV} & S_{VV} \end{bmatrix}$$

Coherency/Covariance Matrix

$$\mathbf{k}_p = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH} + S_{VV} \\ S_{HH} - S_{VV} \\ 2S_{HV} \end{bmatrix}, \mathbf{k}_l = \begin{bmatrix} S_{HH} \\ \sqrt{2}S_{HV} \\ S_{VV} \end{bmatrix}$$

$$\mathbf{T} = \left\langle \mathbf{k}_p \mathbf{k}_p^H \right\rangle$$

$$\mathbf{C} = \left\langle \mathbf{k}_l \mathbf{k}_l^H \right\rangle$$

Second-Order Statistics

Background (2/5)

- Facts of coherency matrix

$$\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{12}^{*} & T_{22} & T_{23} \\ T_{13}^{*} & T_{23}^{*} & T_{33} \end{bmatrix}$$

T_{12} → $\mathrm{Re}(T_{12})$,$\mathrm{Im}(T_{12})$

T_{13} → $\mathrm{Re}(T_{12})$,$\mathrm{Im}(T_{12})$

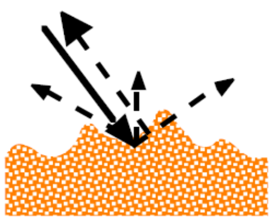
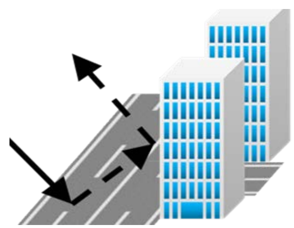
T_{23} → $\mathrm{Re}(T_{12})$,$\mathrm{Im}(T_{12})$

  - Hermitian & Positive-semidefinite
  - 9 independent (real) parameters

Background (3/5)

- Model-Based Decomposition of POLSAR Data
  - Freeman-Durden Decomposition[1]

$$\mathbf{T} = P_S \mathbf{T}_S + P_D \mathbf{T}_D + P_V \mathbf{T}_V$$


$$\mathbf{T}_S = \begin{bmatrix} 1 & \beta^* & 0 \\ \beta & |\beta|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{T}_D = \begin{bmatrix} |\alpha|^2 & \alpha^* & 0 \\ \alpha & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{T}_V = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

  - *Incomplete Utilization of Polarimetric Information: 5 out of 9*
  - *Emergence of Negative Powers: P_S, P_D<0 for certain pixels*

[1] A. Freeman and S. L. Durden, "A three-component scattering model for polarimetric SAR data," *IEEE Trans. Geosci. Remote Sens.*, vol.36, pp.963-973, 1998.

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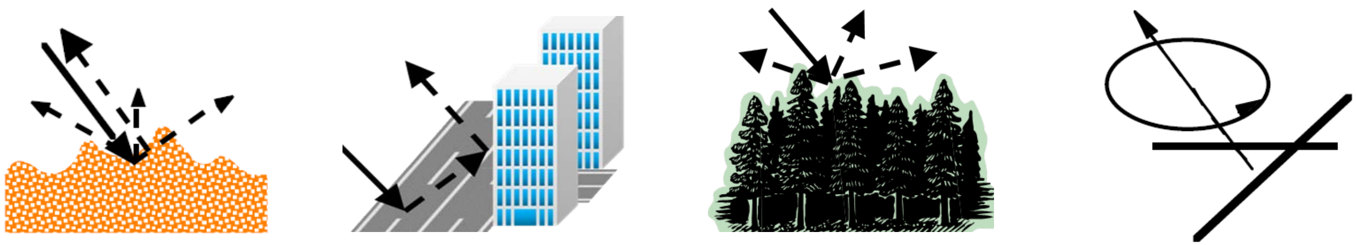
Background (4/5)

- Model-Based Decomposition of POLSAR Data
  - Yamaguchi Decomposition[1]

$$\mathbf{T} = P_S \mathbf{T}_S + P_D \mathbf{T}_D + P_V \mathbf{T}_V + P_C \mathbf{T}_C$$



$$\mathbf{T}_S = \begin{bmatrix} 1 & \beta^* & 0 \\ \beta & |\beta|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{T}_D = \begin{bmatrix} |\alpha|^2 & \alpha^* & 0 \\ \alpha & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{T}_V = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{T}_C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & \pm j \\ 0 & \mp j & 1 \end{bmatrix}$$

  - *Incomplete Utilization of Polarimetric Information: 6 out of 9*
  - *Emergence of Negative Powers: P_S, P_D<0 for certain pixels*

[1] Y. Yamaguchi, T. Moriyama, M. Ishido, and H. Yamada, "Four-component scattering model for polarimetric SAR image decomposition," *IEEE Trans. Geosci. Remote Sens.*, vol. 43, no. 8, pp. 1699-1706, Aug. 2005.

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Background (5/5)

- Timeline of model-based decomposition

1998: three-component decomposition (Freeman et al)

2005: four-component decomposition (Yamaguchi et al)

2010: three-component decomposition with rotation compensation (An et al)
2011: four-component decomposition with rotation compensation (Yamaguchi et al)
2011: non-negative eigenvalue decomposition (van Zyl et al)
2012: four-component decomposition with double transformation(Singh et al)

- Pending issues
  - Prevention of negative powers
  - Complete utilization of full-polarimetric information

Proposed decomposition (1/10)

- **Problem formulation**

$$\mathbf{T} = P_S \mathbf{T}_S + P_D \mathbf{T}_D + P_V \mathbf{T}_V$$

Single Scatterers ($\mathbf{T}_S$, $\mathbf{T}_D$)

Random Volume Scattering ($\mathbf{T}_V$)

- Condition #1: $\mathbf{T}_V$ is a known positive-definite Hermitian matrix
- Condition #2: $\mathbf{T}_S$ and $\mathbf{T}_D$ are unknown **single-rank** matrices
- Condition #3: $P_S, P_D, P_V \geq 0$

Proposed decomposition (2/10)

- Solving for volume scattering power

$$\mathbf{T} = P_S \mathbf{T}_S + P_D \mathbf{T}_D + P_V \mathbf{T}_V \Rightarrow \mathbf{T} - P_V \mathbf{T}_V = P_S \mathbf{T}_S + P_D \mathbf{T}_D$$

$$\left| \mathbf{T} - P_V \mathbf{T}_V \right| = 0$$

- All roots of $|\mathbf{T}-P_V\mathbf{T}_V|=0$ are **non-negative**.
- **Minimum root** is volume scattering power.

Proposed decomposition (3/10)

- Decomposing the Remaining Matrix $\mathbf{T}'=\mathbf{T}-P_V\mathbf{T}_V$

$$\mathbf{T}' = P_S \mathbf{k}_S \mathbf{k}_S^H + P_D \mathbf{k}_D \mathbf{k}_D^H$$

$$\mathbf{k}_S = \begin{bmatrix} \cos\delta_S \\ \sin\delta_S \cos\omega_S e^{j\phi_S} \\ \sin\delta_S \sin\omega_S e^{j\varphi_S} \end{bmatrix} \qquad \mathbf{k}_D = \begin{bmatrix} \cos\delta_D \\ \sin\delta_D \cos\omega_D e^{j\phi_D} \\ \sin\delta_D \sin\omega_D e^{j\varphi_D} \end{bmatrix}$$

- Unknown parameters: **10**
- Available equations from $\mathbf{T}'$: **8**

→ **Underdetermined problem**

Proposed decomposition (4/10)

- Decomposing the Remaining Matrix $\mathbf{T}'=\mathbf{T}-P_V\mathbf{T}_V$
  - **Method 1: Eigendecomposition**[1]

$$\mathbf{T}' = \lambda_1 \mathbf{k}_1 \mathbf{k}_1^H + \lambda_2 \mathbf{k}_2 \mathbf{k}_2^H, \lambda_1 \ge \lambda_2 > 0$$

Dominant Single Scatterer:
Optimal Single-Rank Approximation of $\mathbf{T}'$

$$\mathbf{k}_1 = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$$

$|k_1|>|k_2|$ → Odd-scattering

$|k_1|<|k_2|$ → Even-scattering

[1] J.J. van Zyl, M. Arii, and Y. Kim, "Model-based decomposition of polarimetric SAR covariance matrices constrained for nonnegative eigenvalues," *IEEE Trans. Geosci. Remote Sens.*, vol. 49, no. 9, pp. 1104-1113, Sept. 2011.

Proposed decomposition (5/10)

INPUT: $\mathbf{T}$, $\mathbf{T}_{\mathrm{V}}$

Solve the cubic equation: $\det(\mathbf{T} - x\mathbf{T}_{\mathrm{V}}) = 0 \Rightarrow x_1, x_2, x_3$

Determine the volume scattering power: $P_{\mathrm{V}} = \min\{x_1, x_2, x_3\}, \mathbf{T}' = \mathbf{T} - P_{\mathrm{V}}\mathbf{T}_{V}$

Solve the eigendecomposition problem: $\mathbf{T}' = \lambda_1 \mathbf{k}_1 \mathbf{k}_1^{\mathrm{H}} + \lambda_2 \mathbf{k}_2 \mathbf{k}_2^{\mathrm{H}}$

SWITCH

CASE: $|\mathbf{k}_1(1)| > |\mathbf{k}_1(2)|$ AND $|\mathbf{k}_2(1)| < |\mathbf{k}_2(2)|$

$P_{\mathrm{S}} = \lambda_1, P_{\mathrm{D}} = \lambda_2$

CASE: $|\mathbf{k}_1(1)| < |\mathbf{k}_1(2)|$ AND $|\mathbf{k}_2(1)| > |\mathbf{k}_2(2)|$

$P_{\mathrm{S}} = \lambda_2, P_{\mathrm{D}} = \lambda_1$

CASE: $|\mathbf{k}_1(1)| > |\mathbf{k}_1(2)|$ AND $|\mathbf{k}_2(1)| > |\mathbf{k}_2(2)|$

$P_{\mathrm{S}} = \lambda_1 + \lambda_2, P_{\mathrm{D}} = 0$

CASE: $|\mathbf{k}_1(1)| < |\mathbf{k}_1(2)|$ AND $|\mathbf{k}_2(1)| < |\mathbf{k}_2(2)|$

$P_{\mathrm{S}} = 0, P_{\mathrm{D}} = \lambda_1 + \lambda_2$

END

OUPPUT: $P_{\mathrm{S}}, P_{\mathrm{D}}, P_{\mathrm{V}}$

Proposed decomposition (6/10)

- Decomposing the Remaining Matrix $\mathbf{T}'=\mathbf{T}-P_V\mathbf{T}_V$
  - **Method 2: Optimal Model-Fitting**

$$\mathbf{T}' = P_S\mathbf{T}\left(\beta,\theta\right) + P_D\mathbf{k}_D\mathbf{k}_D^H$$

Rotated Bragg Scatterer

$$\mathbf{T}\left(\beta,\theta\right) = \frac{1}{1+|\beta|^2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & -\sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} 1 & \beta^* & 0 \\ \beta & |\beta|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & -\sin 2\theta \\ 0 & \sin 2\theta & \cos 2\theta \end{bmatrix}$$

Proposed decomposition (7/10)

- **Solution of Optimal Model-Fitting**
  - Step 1: rotation compensation

$$\mathbf{T}'(\theta) = \mathbf{R}^T(\theta)\mathbf{T}'\mathbf{R}(\theta) = \mathbf{R}^T(\theta)\left[P_S\mathbf{T}(\beta,\theta) + P_D\mathbf{k}_D\mathbf{k}_D^H\right]\mathbf{R}(\theta)$$

$$\mathbf{R}^T(\theta)\mathbf{k}_D = \begin{bmatrix} \cos\delta \\ \sin\delta\cos\omega e^{j\phi} \\ \sin\delta\sin\omega e^{j\varphi} \end{bmatrix}$$

$$\mathbf{T}'(\theta) = \frac{P_S}{1+|\beta|^2}\begin{bmatrix} 1 & \beta^* & 0 \\ \beta & |\beta|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$+ P_D\begin{bmatrix} \cos^2\delta & \cos\delta\sin\delta\cos\omega e^{-j\phi} & \cos\delta\sin\delta\sin\omega e^{-j\varphi} \\ \cos\delta\sin\delta\cos\omega e^{j\phi} & \sin^2\delta\cos^2\omega & \sin^2\delta\cos\omega\sin\omega e^{j(\phi-\varphi)} \\ \cos\delta\sin\delta\sin\omega e^{j\varphi} & \sin^2\delta\cos\omega\sin\omega e^{j(\varphi-\phi)} & \sin^2\delta\sin^2\omega \end{bmatrix}$$

Proposed decomposition (8/ 10)

- **Solution of Optimal Model-Fitting**
  - Step 2: solving unknowns in terms of $\mathbf{T}'(\theta)$

$$\mathbf{T}'(\theta)=\frac{P_S}{1+|\beta|^2}\begin{bmatrix}1 & \beta^* & 0\\ \beta & |\beta|^2 & 0\\ 0 & 0 & 0\end{bmatrix}+P_D\begin{bmatrix}\cos^2\delta & \cos\delta\sin\delta\cos\omega e^{-j\phi} & \cos\delta\sin\delta\sin\omega e^{-j\varphi}\\ \cos\delta\sin\delta\cos\omega e^{j\phi} & \sin^2\delta\cos^2\omega & \sin^2\delta\cos\omega\sin\omega e^{j(\phi-\varphi)}\\ \cos\delta\sin\delta\sin\omega e^{j\varphi} & \sin^2\delta\cos\omega\sin\omega e^{j(\varphi-\phi)} & \sin^2\delta\sin^2\omega\end{bmatrix}$$

$$P_S=T'_{11}(\theta)+T'_{22}(\theta)-\frac{|T'_{13}(\theta)|^2+|T'_{23}(\theta)|^2}{T'_{33}(\theta)}$$

$$\beta=\frac{T'_{33}(\theta)[T'_{12}(\theta)]^*-T'_{23}(\theta)[T'_{13}(\theta)]^*}{T'_{11}(\theta)T'_{33}(\theta)-|T'_{13}(\theta)|^2}$$

$$P_D=T'_{33}(\theta)+\frac{|T'_{13}(\theta)|^2+|T'_{23}(\theta)|^2}{T'_{33}(\theta)}$$

$$\delta=\sin^{-1}\left[\sqrt{\frac{|T'_{33}(\theta)|^2+|T'_{23}(\theta)|^2}{|T'_{33}(\theta)|^2+|T'_{23}(\theta)|^2+|T'_{13}(\theta)|^2}}\right]$$

$$\omega=\sin^{-1}\left[\sqrt{\frac{|T'_{33}(\theta)|^2}{|T'_{33}(\theta)|^2+|T'_{23}(\theta)|^2}}\right]$$

$$\phi=\arg\{T'_{23}(\theta)[T'_{13}(\theta)]^*\}$$

$$\varphi=-\arg[T'_{13}(\theta)]$$

For any θ, P_S, $P_D \geq 0$

Proposed decomposition (9/10)

Solution of Optimal Model-Fitting

- Step 3: maximizing P_S with respect to θ

$$\max_{\theta}\left\{T'_{11}(\theta)+T'_{22}(\theta)-\frac{|T'_{13}(\theta)|^2+|T'_{23}(\theta)|^2}{T'_{33}(\theta)}\right\} \Rightarrow \theta_{\max}=\frac{1}{4}\left[\tan^{-1}\frac{\left(B+\sqrt{B^2-C}\right)d-a}{\left(B+\sqrt{B^2-C}\right)e-b}+k\pi\right]$$

$$\left\{\begin{aligned}
B &= \frac{ad+eb-cf}{d^2+e^2-f^2}\\
C &= \frac{a^2+b^2-c^2}{d^2+e^2-f^2}\\
a &= \frac{1}{2}\left[T'_{11}T'_{33}-|T'_{13}|^2-T'_{11}T'_{22}+|T'_{12}|^2\right]\\
b &= T'_{11}\mathrm{Re}(T'_{23})-\mathrm{Re}\left[T'_{12}(T'_{13})^*\right]\\
c &= T'_{22}T'_{33}-|T'_{23}|^2+\frac{1}{2}\left(T'_{11}T'_{33}-|T'_{13}|^2+T'_{11}T'_{22}-|T'_{12}|^2\right)\\
d &= \frac{1}{2}(T'_{33}-T'_{22})\\
e &= \mathrm{Re}(T'_{23})\\
f &= \frac{1}{2}(T'_{22}+T'_{33})
\end{aligned}\right\}$$

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Proposed decomposition (10/10)

INPUT: $\mathbf{T},\ \mathbf{T}_\mathrm{V}$

Solve the cubic equation: $\det(\mathbf{T} - x\mathbf{T}_\mathrm{V}) = 0 \Rightarrow x_1, x_2, x_3$

Detemine the volume scattering power: $P_V = \min\{x_1, x_2, x_3\}, \mathbf{T}' = \mathbf{T} - P_V \mathbf{T}_V$

Detemine the constants a, b, c, d, e, f, B, C

Obtain the rotation angle: $\theta = \frac{1}{4}\left[\tan^{-1}\frac{(B+\sqrt{B^2-C})d-a}{(B+\sqrt{B^2-C})e-b} + k\pi\right]$

Rotate the remaining matrix: $\mathbf{T}'(\theta) = \mathbf{R}^\mathrm{T}(\theta)\mathbf{T}'\mathbf{R}(\theta)$

$|\beta| = \left|\frac{T'_{33}(\theta)[T'_{12}(\theta)]^* - T'_{13}(\theta)[T'_{23}(\theta)]^*}{T'_{11}(\theta)T'_{33}(\theta) - |T'_{13}(\theta)|^2}\right|, |\tan\delta\cos\omega| = \left|\frac{T'_{23}(\theta)}{T'_{13}(\theta)}\right|$

SWITCH

CASE: $|\beta| < 1$ **AND** $|\tan\delta\cos\omega| > 1$

$P_S = T'_{11}(\theta) + T'_{22}(\theta) - \frac{|T'_{13}(\theta)|^2 + |T'_{23}(\theta)|^2}{T'_{33}(\theta)}, P_D = T'_{33}(\theta) + \frac{|T'_{13}(\theta)|^2 + |T'_{23}(\theta)|^2}{T'_{33}(\theta)}$

CASE: $|\beta| > 1$ **AND** $|\tan\delta\cos\omega| < 1$

$P_S = T'_{33}(\theta) + \frac{|T'_{13}(\theta)|^2 + |T'_{23}(\theta)|^2}{T'_{33}(\theta)}, P_D = T'_{11}(\theta) + T'_{22}(\theta) - \frac{|T'_{13}(\theta)|^2 + |T'_{23}(\theta)|^2}{T'_{33}(\theta)}$

CASE: $|\beta| < 1$ **AND** $|\tan\delta\cos\omega| < 1$

$P_S = T'_{11}(\theta) + T'_{22}(\theta) + T'_{33}(\theta), P_D = 0$

CASE: $|\beta| > 1$ **AND** $|\tan\delta\cos\omega| > 1$

$P_S = ,0\ P_D = T'_{11}(\theta) + T'_{22}(\theta) + T'_{33}(\theta)$

END

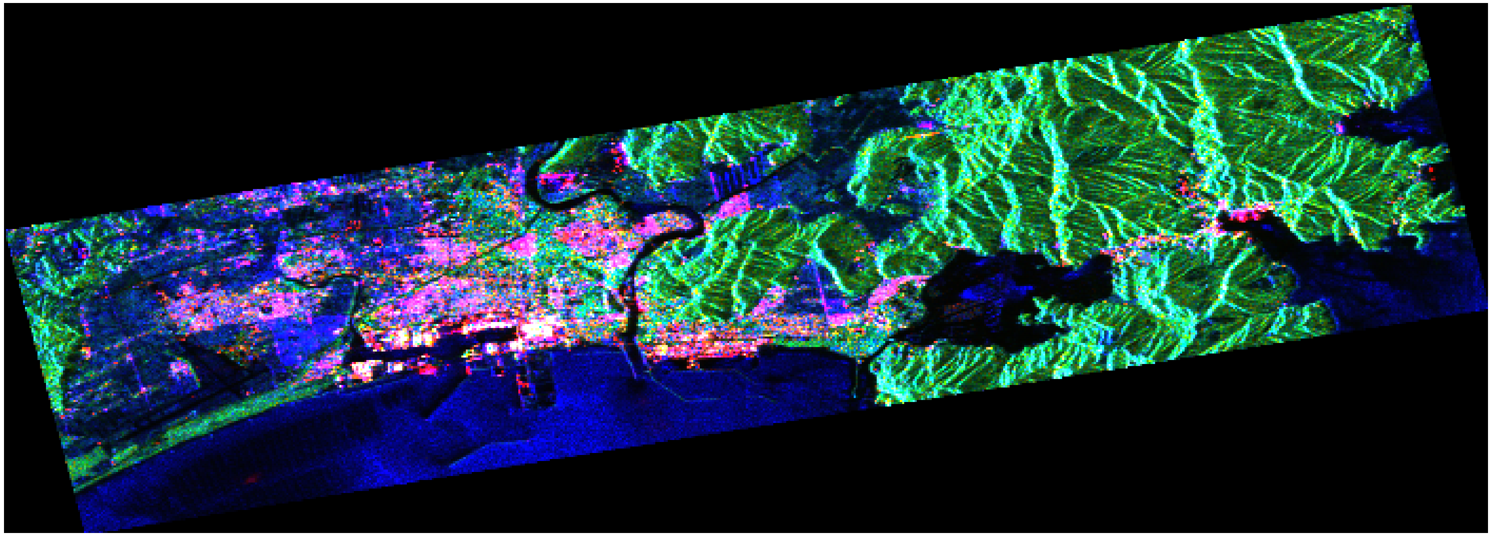
OUPPUT: P_S, P_D, P_V

Application to post-tsunami area (1/4)

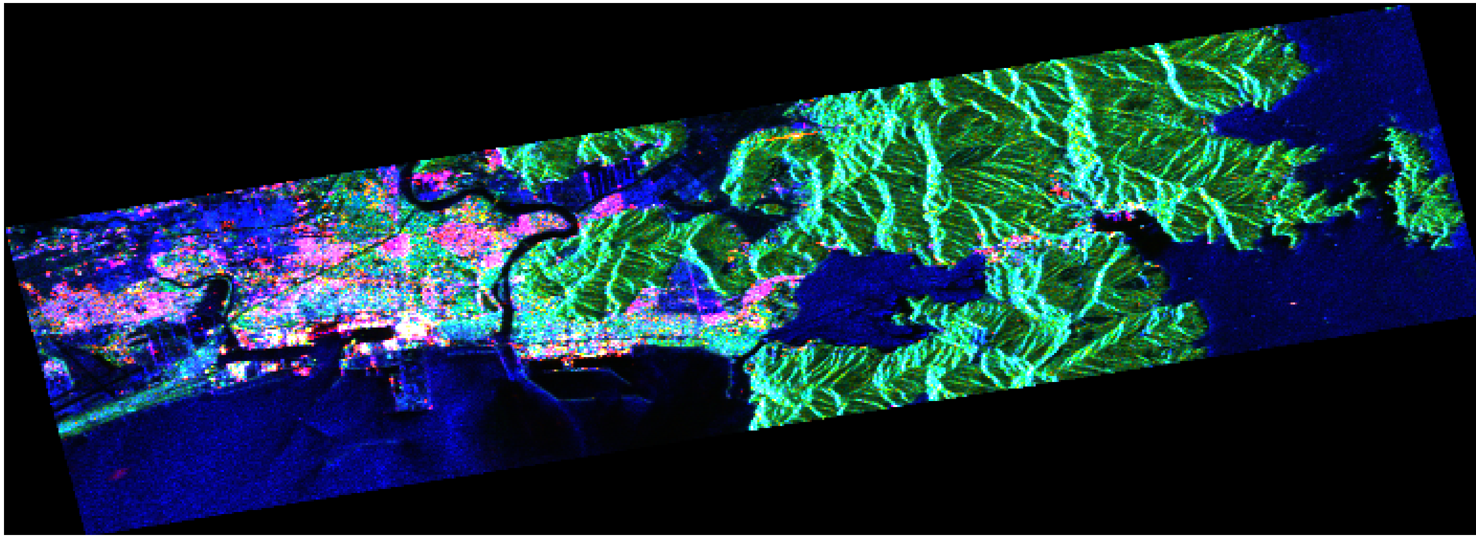
- Experimental data
  - **Sensor**: ALOS-PALSAR
  - **Mode**: fully-polarimetric
  - **Location**: Ishinomaki area
  - **Data**:
    - ALPSRP257090760-P1.1__A (Nov. 21, 2010, before the earthquake)
    - ALPSRP277220760-P1.1__A (Apr. 8, 2011 after the earthquake,).
- Preprocessing
  - Reprojection to the World Geodetic System (WGS 84).
  - Spatial multi-look processing with 9 × 9 pixels[1].

[1] J.-S. Lee, T.L. Ainsworth, J.P. Kelly, and C. Lopez-Martinez, "Evaluation and bias removal of multilook effect on entropy/alpha/anisotropy in polarimetric SAR decomposition," *IEEE Trans. Geosci. Remote Sens.*, vol. 46, no. 10, pp. 3039-3052, Oct. 2008.

Application to post-tsunami area (2/4)



Nov. 21, 2010

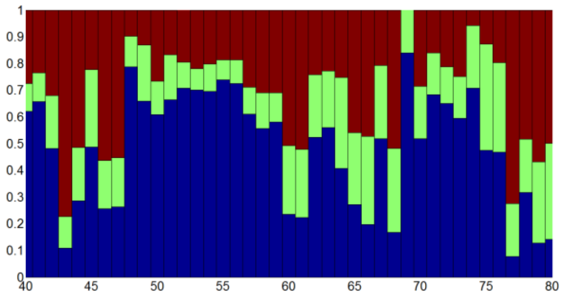


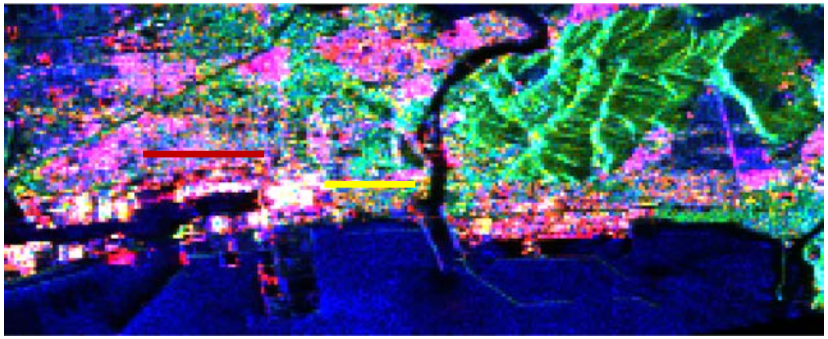
Apr. 08, 2011

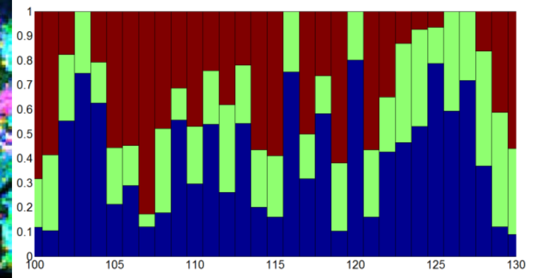
- double-boucne
- volume
- surface

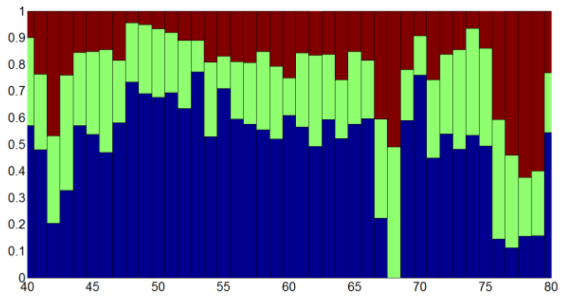
Application to post-tsunami area (3/4)

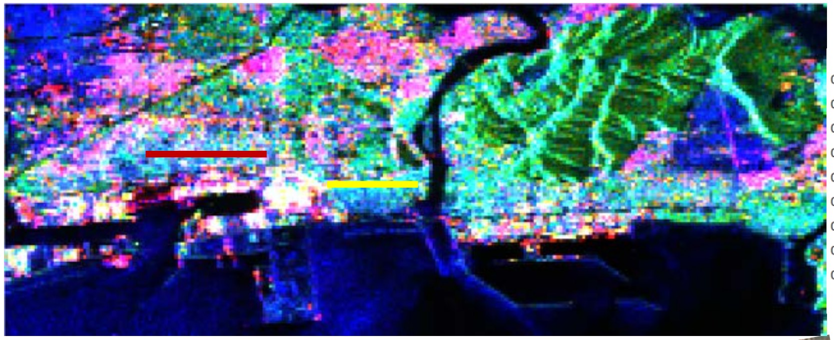
- Ishinomaki City

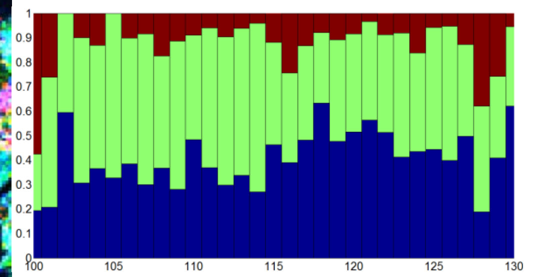






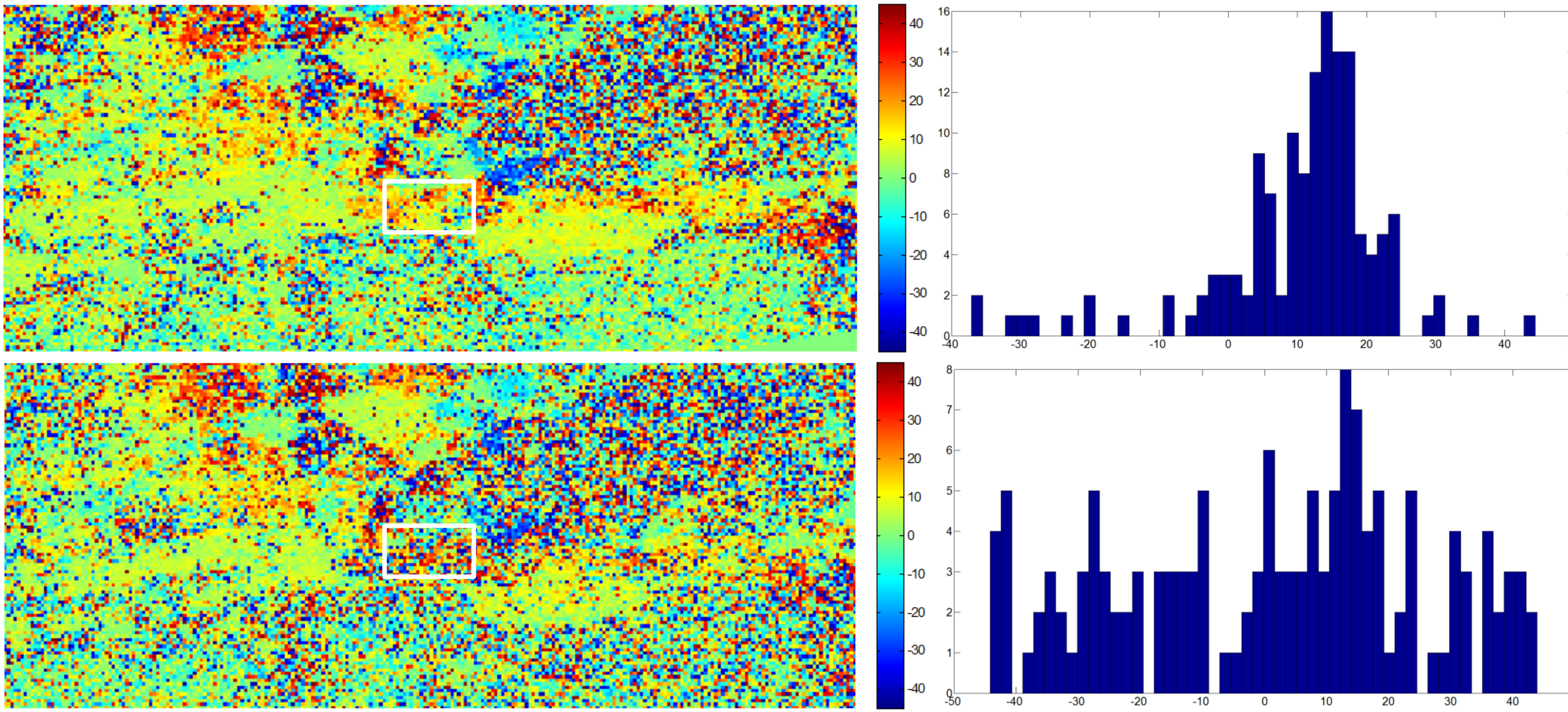






Application to post-tsunami area (4/4)

- Orientation angle



Conclusion

- **New scattering power decomposition**
  - Measured coherency matrix is assumed to be the combined contribution of volume scattering and two single scatterers.
  - Volume scattering power is uniquely equal to the minimum root of $|\mathbf{T}\text{-}P_V\mathbf{T}_V|=0$.
  - Methods to discriminate two single scatterers.
    - *Eigendecomposition method*
    - *Optimal model fitting method*
- **Preliminary results in post-tsunami analysis**
  - Volume scattering power tends to increase in tsunami affected area.
  - In urban areas, double-bounce scattering power tends to decrease due to the destruction of man-made structures.
  - Surface scattering power tends to increase in totally washed-out area[1].
  - Orientation angle tends to randomize in tsunami affected area[2].

[1] Y. Yamaguchi, "Disaster monitoring by fully polarimetric SAR data acquired with ALOS-PALSAR," *Proceedings of IEEE*, vol. 100, no. 10, Oct. 2012.

[2] S.-W. Chen and M. Sato, "Tsunami damage investigation of built-up areas using multitemporal spaceborne fully polarimetric SAR images," *IEEE Trans. Geosci. Remote Sens.*, in press.

THANK YOU!